

Fluid Mechanics and Machinery

Mechanics:

Fluid Mechanics is that branch of Science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion.

Study of fluid at rest $\rightarrow$ Fluid statics

Study of fluid in motion } $\rightarrow$ Fluid Kinematics
Pressure force not considered

Applications

R&AC

Pump & motors

hy braking sys

Turbine

Air comp

Hyd damp

Study of fluid in motion } $\rightarrow$ Fluid dynamics
with Pressure force considered

Fluid $\rightarrow$ (exp) water, kerosene, milk, gasoline, air, steam etc.
Liquid & gas together are called fluids.

Fluid may be defined as a substance which is capable of flowing.

It has no definite shape of its own, but conforms to the shape of the containing vessel.

Generally, matter exists in three states.

They are

i) Solid

ii) Liquid

iii) Gas.

Fluid has the following P

* It ~~has~~ flows under its own weight and takes the shape of any solid both with which it is contained.

* Shear stresses $\rightarrow$ stress is parallel to the surface of the material. Shear stresses occur in fluids only when they are in motion.

* A fluid in equilibrium cannot sustain any shear stress.

Distinction between Solid and fluid:

✓ Solid has its own shape and undergoes an infinitesimal change in volume under compressive load. [Fluids do not have their own shape].

Intermolecular bonds are very strong in solids, weak in liquids and very weak in gases.

Classification of fluids:

- i) Ideal fluids and Real or Practical fluids
- ii) Newtonian fluids and Non-Newtonian fluids

1. Ideal fluid:

✓ Ideal fluids have the following properties:

- i) It is incompressible [water]
- ii) It has zero viscosity
- iii) Shear stress is zero when the fluid is in motion. [No resistance is offered to the motion of any fluid]

2. Real or Practical fluids:

- i) It is compressible (example: air)
- ii) They are viscous in nature
- iii) Some resistance is always offered by the fluid when it is in motion.
- iv) shear stress always exists in such fluids.

3. Newtonian fluids:

A linear relationship exists between the magnitude of shear stress (τ) and the resulting rate of deformation (du/dy). The constant of proportionality μ does not change with the rate of deformation. [Newton's law of viscosity]

$$\tau = \mu \frac{du}{dy}$$

It obeys Newton's law of viscosity.

Eg: water, kerosene

viscosity @ any given temp & pressure is constant.

4. Non-Newtonian fluids:

It does not obey the Newton's law of viscosity.

non-linear relationship b/w

Eg: Paint, toothpaste, Printer's ink

5. Ideal plastic fluid:

Fundamental Dimensions and Units:

All physical quantities are measured in certain units.

Two types

1. Fundamental units
2. Derived units.

1. Fundamental Units:

All the physical quantities are expressed in terms of the following three fundamental units:

1. Length (L)

2. Mass (M)

3. Time (T)

2. Derived Units:

Some units called derived units are expressed in terms of fundamental units, such as units of area, velocity, acceleration, Pressure etc.

m^2

m/s

m/s^2

(01)

~~mm~~

N/m^2

Fluid Properties:

3

1) Mass density (ρ): $PV = mRT$

Mass density or Density is defined as the mass per unit Volume i.e., mass of fluid contained in 1m^3 volume.

(rho) Density $\rho = \frac{\text{mass}}{\text{Volume}} = \frac{m}{V} \text{ Kg/m}^3$

mass $\rho = \frac{\text{specific weight}}{\text{Acceleration due to gravity}} = \frac{w}{g} \text{ kg/m}^3$

Density $\downarrow$ with increase in temperature
Density $\uparrow$ with increase in pressure ($g = 9.81 \text{ m/s}^2$)

✓ water density 1000 kg/m^3

2) Specific Volume (v):

It is defined as volume of fluid occupied by a unit mass or Volume per unit mass of a fluid is called specific volume.

Specific volume $v = \frac{\text{Volume}}{\text{mass}} = \frac{V}{m} \text{ m}^3/\text{kg}$ (or) $\frac{1}{\text{Density}} = \frac{1}{\rho}$

3) Specific weight (w):

Weight possessed per unit Volume.

Specific weight, $w = \frac{\text{Weight}}{\text{Volume}} = \frac{W}{V} = \frac{mg}{V} = \rho g$

4) Specific gravity (S):

$S = \frac{\text{Specific weight of a given fluid}}{\text{Specific weight of the std fluid}}$

$\downarrow$
 9.81 N/m^3
of water

(or) $S = \frac{\text{Mass density of the given fluid}}{\text{Mass density of the std fluid}}$

density of liquid
density of water

$$\left. \begin{array}{l} \text{Specific weight or weight density} \\ \text{of a liquid} \end{array} \right\} = \left[\begin{array}{l} \text{Specific gravity} \\ \text{of liquid} \end{array} \right] \times \left[\begin{array}{l} \text{Specific weight} \\ \text{of water} \end{array} \right]$$

$$= S \times 9.81 \text{ N/m}^3 \Rightarrow \frac{W \rightarrow N}{V \rightarrow \text{m}^3} = \frac{9.81 \times 1000}{\text{m}^3}$$

$$\text{Density of the given liquid} = \left[\begin{array}{l} \text{Specific gravity} \\ \text{of liquid} \end{array} \right] \times \left[\begin{array}{l} \text{Density of} \\ \text{water} \end{array} \right]$$

density

$$\rho = S \times 1000 \text{ kg/m}^3$$

Problems

- 1) Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7 N.

Given:

$$\text{Volume} = 1 \text{ litre} = 1 \times 10^{-3} \text{ m}^3 \quad \left[1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ (or) } 1000 \text{ cm}^3 \right]$$

$$\text{weight} = 7 \text{ N}$$

Sol:

$$\text{i) Specific weight } (\gamma) = \frac{\text{weight}}{\text{Volume}} = \frac{7 \text{ N}}{1 \times 10^{-3}} = 7000 \text{ N/m}^3$$

$$\text{ii) Density } (\rho) = \frac{\overset{\text{sp. weight}}{W}}{\underset{\text{gravity}}{g}} = \frac{7000}{9.81} = 713.5 \text{ kg/m}^3$$

$$\Rightarrow \frac{\frac{\text{N}}{\text{m}^3}}{\frac{\text{m}}{\text{s}^2}} \Rightarrow \frac{\text{N}}{\text{m}^3} \times \frac{\text{s}^2}{\text{m}} = \frac{\text{kg} \cdot \text{m/s}^2 \cdot \text{s}^2}{\text{m}^4} = \frac{\text{kg}}{\text{m}^3}$$

$$\text{iii) Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} = 0.7135$$

no unit

2. Calculate the density, specific weight and weight, of one litre of petrol of specific gravity = 0.7.

Given:

$$\text{Volume} = 1 \text{ litre} = 1 \times 10^{-3} \text{ m}^3 \quad (\text{or}) \quad \frac{1}{1000} \text{ m}^3$$

$$\text{Specific gravity } S = 0.7 = \frac{\text{mass of given fluid}}{\text{mass of standard fluid}}$$

Solution:

i) Density $\rho = S \times 1000 = 0.7 \times 1000 = 700 \text{ kg/m}^3$

ii) Specific weight (w) = $\rho \times g = 700 \times 9.81 = 6867 \text{ N/m}^3$

iii) weight (w) $\Rightarrow \frac{\text{Specific } (w)}{\text{Weight}} = \frac{\text{Weight } (w)}{\text{Volume}}$

$$6867 = \frac{w}{1 \times 10^{-3}}$$

$$\boxed{w = 6.867 \text{ N}}$$

3. Calculate the specific weight, mass density, specific gravity and specific volume of oil having volume of 4.5 m^3 and weight of 40 kN .

Given:

$$\text{Volume of oil} = 4.5 \text{ m}^3$$

$$\text{Weight of oil} = 40 \text{ kN} = 40 \times 10^3 \text{ N}$$

Sol:

$$\begin{aligned} \text{1. Specific weight } (w) &= \frac{\text{Weight of oil}}{\text{Volume of oil}} = \frac{w}{V} = \frac{40 \times 10^3}{4.5} \frac{\text{N}}{\text{m}^3} \\ &= 8.889 \times 10^3 \text{ N/m}^3 \end{aligned}$$

$$2. \text{Mass density of oil } (\rho) = \frac{\text{Specific weight of oil}}{\text{Acceleration due to gravity}} = \frac{w}{g}$$

$$\frac{\text{kg} \times \text{m}}{\text{m}^3 \times \text{s}^2} = \frac{\text{kg}}{\text{m}^3}$$

$$\frac{\text{m}}{\text{s}^2}$$

$$= \frac{8.889 \times 10^3}{9.81} = 906.1 \text{ kg/m}^3$$

$$3. \text{Specific gravity of oil } (s) = \frac{\text{Specific weight of oil}}{\text{Specific weight of water}}$$

$$\rho_g = \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}}{\text{s}^2} = \frac{\text{KN}}{\text{m}^3}$$

$$\text{Specific weight of water} \rightarrow$$

$$\rho_g$$

$$1000 \times 9.81$$

$$N = \frac{\text{kgm}}{\text{s}^2}$$

$$9.81 \text{ KN/m}^3$$

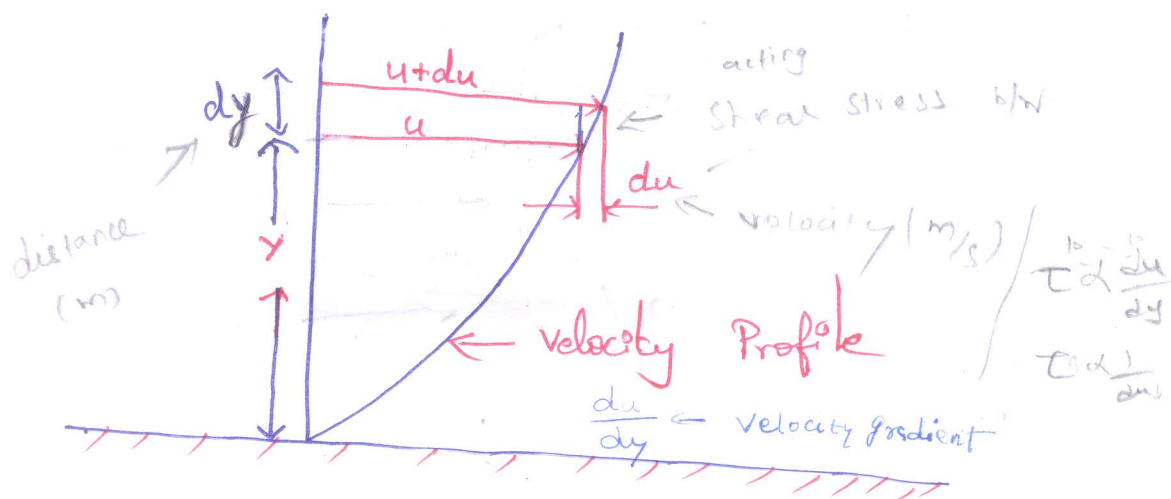
$$= \frac{8.889 \times 10^3 \text{ N/m}^3}{9.81 \times 10^3 \frac{\text{N}}{\text{m}^3}} = 0.906$$

$$4. \text{Specific Volume of oil } v = \frac{1}{\rho} = \frac{1}{906.1} = 1.1 \times 10^{-3} \text{ m}^3/\text{kg}$$

Viscosity:

It is an internal property of a fluid that offers resistance to flow.

viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.



When two layers of a fluid, a distance ' dy ' apart, move one over the other at different velocities, say u and $u+du$ as shown.

The viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to y .

1. Newton's law of Viscosity:

It states that the shear stress τ on the fluid element layer is directly proportional to the rate of shear strain.

The constant of Proportionality is called coefficient of viscosity.

Diagram illustrating shear stress and strain. A fluid element of thickness dy is shown. Shear stress τ is applied to the top and bottom surfaces, causing a shear strain γ . The velocity gradient is $\frac{du}{dy}$. The shear stress τ is related to the velocity gradient by the equation:

$$\tau = \mu \cdot \frac{du}{dy}$$

where μ is the coefficient of viscosity. The unit of τ is N/m^2 .

Fluid obeys Newton's law of viscosity is known as Newtonian fluids.

2. Dynamic Viscosity (μ):

Shear stress required causing unit rate of shear deformation.

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)} \quad \frac{N \cdot s}{m^2} \text{ (SI unit)} \quad \left| \quad \frac{N/m^2}{\frac{m}{s \cdot m}} \Rightarrow \frac{N \cdot s}{m^2}$$

3. Kinematic viscosity (ν):

The kinematic viscosity is defined as the ratio of dynamic viscosity to mass density.

$$\nu = \frac{\mu}{\rho} \quad \frac{m^2}{s} = \frac{N \cdot s}{m^2} \cdot \frac{kg/m^3}{kg/m^3} = \frac{N \cdot s}{kg \cdot m} = \frac{kg \cdot m/s^2 \cdot s}{kg \cdot m} = \frac{m^2}{s}$$

1 Stoke = $\frac{1}{100}$ cm²/s = 10^{-4} m²/s
1 centistoke = $\frac{1}{100}$ Stoke = 10^{-6} m²/s

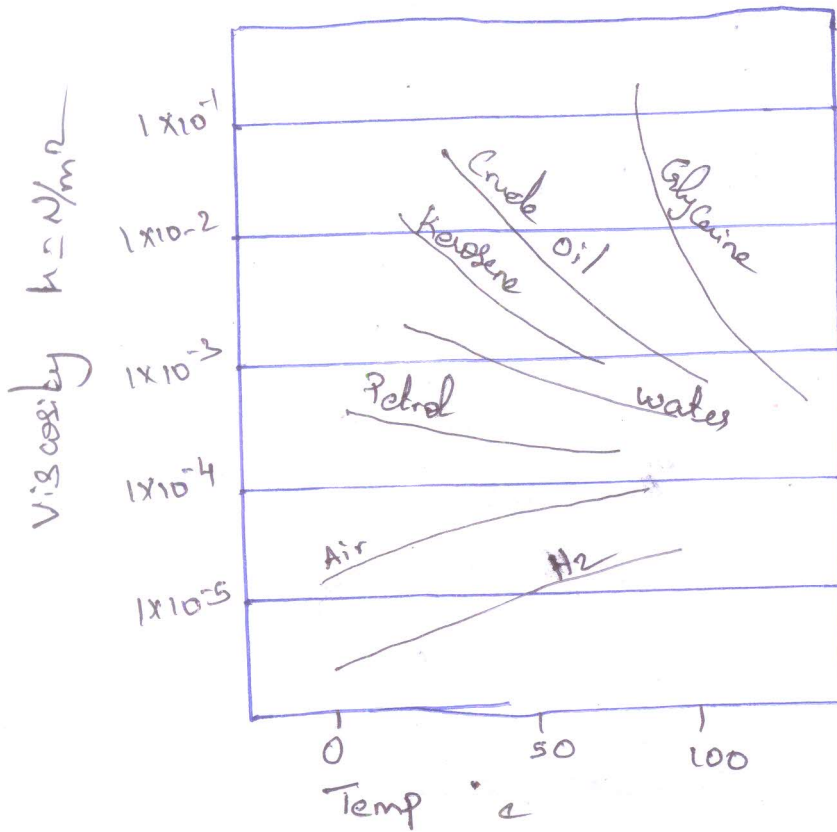
Effect of temperature on viscosity:

6

(Formulae)

$$\text{Power} = F \times u \text{ (Watt)}$$

$$\frac{F}{A} = \tau$$



- 1) Calculate the ^{Dynamic} viscosity of a liquid having kinematic viscosity of 8.5 Stokes and specific gravity 1.4.

Given data: To find: Dynamic viscosity (μ) 1 Stokes = $10^{-4} \text{ m}^2/\text{s}$

Kinematic viscosity, $\nu = 8.5 \text{ Stokes} = 8.5 \times 10^{-4} \text{ m}^2/\text{s}$

Specific gravity $S = 1.4$.

Sol: Solution:

$$1. \quad S = \frac{\text{Density of liquid}}{\text{Density of water}} \quad \Rightarrow \quad 1.4 = \frac{\text{Density of liquid}}{1000}$$

(Specific gravity)

Density of liquid

$$\rho = 1000 \times 1.4 = 1400 \text{ kg/m}^3$$

$$S \times 1000 = \rho \quad \left(\begin{array}{l} \text{sp. gravity of liquid} \times \text{density of water} \\ \leftarrow \end{array} \right)$$

2. Kinematic viscosity:

$$\nu = \frac{\mu}{\rho}$$

$$8.5 \times 10^{-4} = \frac{\mu}{1400}$$

$$\mu = 1.19 \text{ N}\cdot\text{s}/\text{m}^2$$

$$\tau = \mu \times \frac{du}{dy}$$

$$\mu = \tau \times \frac{dy}{du} = \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}}{\frac{\text{m}}{\text{s}}} = \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

2) The velocity ^{glycerin} gradient at a certain point of stream of glycerin is 0.32 s^{-1} . The density of the glycerin is $1270 \text{ kg}/\text{m}^3$ and the kinematic viscosity is $6.5 \times 10^{-6} \text{ m}^2/\text{s}$. Calculate the shear stress at the point

Given data: [Difference in velocity between adjacent layers] = $\text{m} \times \text{s}^{-1}$
 $\uparrow$
 velocity gradient, $(\frac{du}{dy}) = 0.32 \text{ s}^{-1}$
 $\text{N} = \text{kg} \cdot \text{m}/\text{s}^2$

Density, $\rho = 1270 \text{ kg}/\text{m}^3$

Kinematic viscosity, $\nu = 6.5 \times 10^{-6} \text{ m}^2/\text{s}$
 $\text{Nm} = \text{kg} \cdot \text{m}^2/\text{s}^2$

Sol:

Kinematic viscosity, $\nu = \frac{\mu}{\rho}$

$$\mu = \rho \times \nu = 1270 \times 6.5 \times 10^{-6}$$

$$= 8.255 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$$

Shear stress, $\tau = \mu \times \frac{du}{dy}$

$$= 8.255 \times 10^{-3} \times 0.32$$

$$= 2.64 \times 10^{-3} \text{ N}/\text{m}^2$$

$$\frac{\text{kg} \times \text{s}}{\text{m} \cdot \text{s} \cdot \text{s}} \Rightarrow \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = \frac{\text{N}}{\text{m}^2}$$

$$\frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$\tau = \mu \times \frac{du}{dy}$$

3) The space between two parallel plates 0.5 mm apart is filled with mineral oil. A force of 15 N is required to pull the upper plate at a constant velocity of 0.6 m/s. Calculate the viscosity of oil, if the area of the upper plate is $2 \times 10^6 \text{ mm}^2$. Find also the kinematic viscosity of the oil if its relative density is 0.8.

Given data:

Distance between the plate $dy = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$

Force on upper plate $F = 15 \text{ N}$

velocity of the plate

over other $du = 0.6 \text{ m/s}$

Area of upper plate, $A = 2 \times 10^6 \text{ mm}^2$
 $= 2 \text{ m}^2$

Relative density (or)

specific gravity (s) = 0.8

Sol:

shear stress, $\tau = \frac{F}{A} = \frac{15}{2} = 7.5 \text{ N/m}^2$

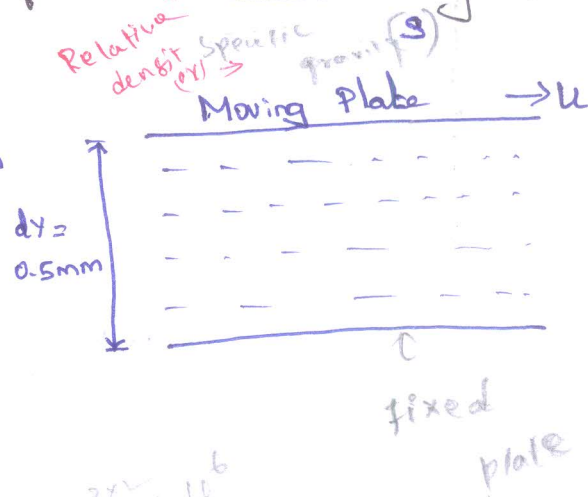
$\tau = \mu \times \frac{du}{dy} \Rightarrow 7.5 = \mu \times \frac{0.6}{0.5 \times 10^{-3}}$

$\mu = 6.25 \times 10^{-3} \frac{\text{N}\cdot\text{s}}{\text{m}^2} = 6.25 \times 10^{-2} \text{ Poise}$

Density of oil, $\rho = s \times \text{Density of water}$

$= 0.8 \times 1000 = 800 \text{ kg/m}^3$

Kinematic viscosity $\nu = \frac{\mu}{\rho} = \frac{6.25 \times 10^{-3}}{800} = 7.81 \times 10^{-6} \text{ m}^2/\text{s}$



To find:
kinematic viscosity (or)

$\text{N} = \text{kg} \cdot \text{m/s}^2$
 $\text{kg} \times \frac{\text{m}}{\text{s}^2}$

$\frac{\text{N}\cdot\text{s}}{\text{m}^2} = \frac{\text{kg} \cdot \text{m/s}^2 \cdot \text{s}}{\text{m}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s} \cdot \text{m}^2} = \frac{\text{kg}}{\text{s}}$

4) Two large inclined parallel planes are kept 8mm apart and are filled with oil. The parallel planes are inclined at 30° to the horizontal. A small thin square plate of 75mm side slides freely down parallel and midway between the inclined planes with a constant velocity 2.5 m/s due to its own weight of 1.5N. Determine the viscosity of the oil.

G.D:

Distance between parallel plates, $dy = 8\text{mm} = 8 \times 10^{-3}$

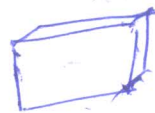
Angle of inclination, $\alpha = 30^\circ$

Side of the plate = 75mm = 0.075m

Area of the plate = $0.075 \times 0.075 \text{ m}^2$

Velocity of the plate, $du = 2.5 \text{ m/s}$

Weight of the plate, $W = 1.5 \text{ N}$

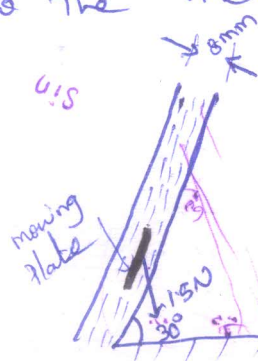
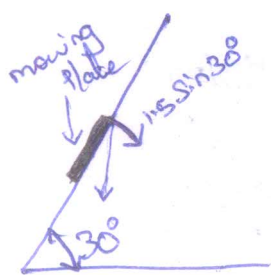


Sol:

The vertical force of 1.5N due to weight of the plate can be resolved into two components.

One is along inclined line

other is $\perp$ to the inclined plane



(8)

Force due to the weight of moving plate
along the direction of motion,

$$F = 1.5 \times \sin 30^\circ = 1.5 \times \cos 60^\circ \\ = 0.75 \text{ N}$$

Since oil is contacting both side of the
plate, the total area on which the oil
resist. 1

viscous force $A = 2 \times \text{Area of the plate}$

$$= 2 \times 0.075 \times 0.075 \\ = 0.01125 \text{ m}^2$$

shear stress acting on the plate

$$\tau = \frac{F}{A} = \frac{0.75}{0.01125} = 66.67 \text{ N/m}^2$$

shear stress, $\tau = \mu \times \frac{du}{dy}$

$$66.67 = \mu \times \frac{2.5}{8 \times 10^{-3}}$$

dynamic
viscosity

$$\mu = 0.213 \text{ N-s/m}^2$$

5) A hydraulic lift shaft of 225mm dia moves in a cylinder of 227mm dia with the length of engagement of 1.2m. The interface is filled with oil of kinematic viscosity of $3.4 \times 10^{-4} \text{ m}^2/\text{s}$ and density 950 kg/m^3 . Determine the uniform velocity of movement of the shaft, if the drag resistance was 480 N. (fluid friction)

Given Data:

Shaft diameter, $D_s = 225 \text{ mm} = 0.225 \text{ m}$

Cylinder diameter, $D_c = 227 \text{ mm} = 0.227 \text{ m}$

length of engagement, $l = 1.2 \text{ m}$

kinematic viscosity of oil $\nu = 3.4 \times 10^{-4} \text{ m}^2/\text{s}$

Density of oil $\rho = 950 \text{ kg/m}^3$

Drag resistance, $F = 480 \text{ N}$

Solution:

Area of contact of oil with shaft,

$$A = \pi \times D_s \times l = \pi \times 0.225 \times 1.2 = 0.848 \text{ m}^2$$

clearance between cylinder and shaft

$$dy = \frac{227 - 225}{2} = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

shear stress,

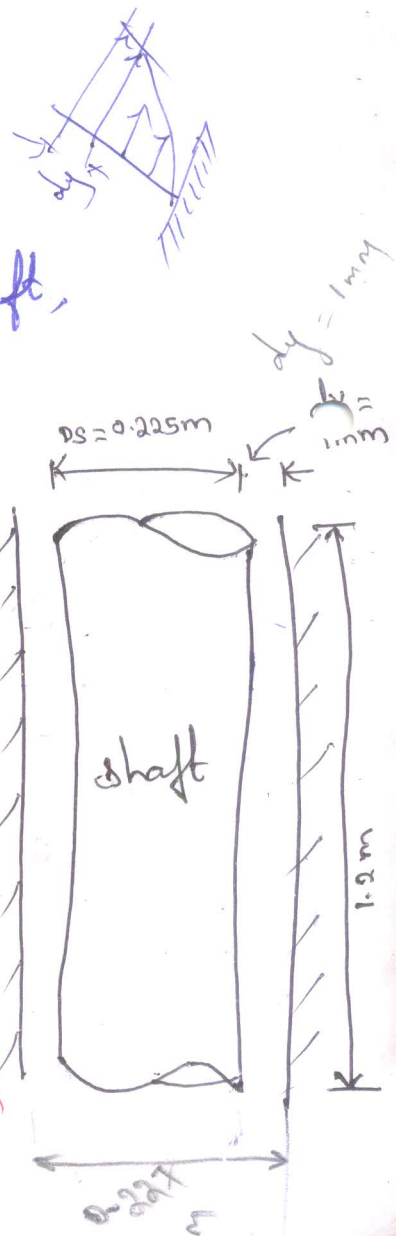
$$\tau = \frac{F}{A} = \frac{480}{0.848} = 566.04 \text{ N/m}^2$$

$$\text{kinematic viscosity } \nu = \frac{\mu}{\rho} \Rightarrow 3.4 \times 10^{-4} = \frac{\mu}{950} \Rightarrow \mu = 0.323 \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

W.K.T shear stress

$$\tau = \mu \times \frac{du}{dy} \Rightarrow 566.04 = 0.323 \times \frac{du}{0.001} = \frac{566.04 \times du}{0.323}$$

$$u = du = 1.75 \text{ m/s}$$



6) The velocity distribution for air flow over a flat plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in metre per second at a distance y metre above the plate. Determine the shear stress at $y=0$ and $y=0.15$ m. Take dynamic viscosity of fluid as 8.63 poise.

Given:

$$u = \frac{2}{3}y - y^2$$

Dynamic viscosity

$$\mu = 8.63 \text{ Poise} = 0.863 \text{ N-s/m}^2$$

Sol:

$$\text{Velocity distribution, } u = \frac{2}{3}y - y^2$$

$$\text{Velocity gradient, } \frac{du}{dy} = \frac{2}{3} - 2y$$

We know that

$$\tau = \mu \frac{du}{dy} = 0.863 \times \left(\frac{2}{3} - 2y \right)$$

Case (i): Shear stress at $y=0$

$$\tau = 0.863 \times \left(\frac{2}{3} - (2 \times 0) \right) = 0.575 \text{ N/m}^2$$

Case (ii): Shear stress at $y=0.15$ m

$$\tau = 0.863 \times \left(\frac{2}{3} - 2 \times 0.15 \right) = 0.316 \text{ N/m}^2$$

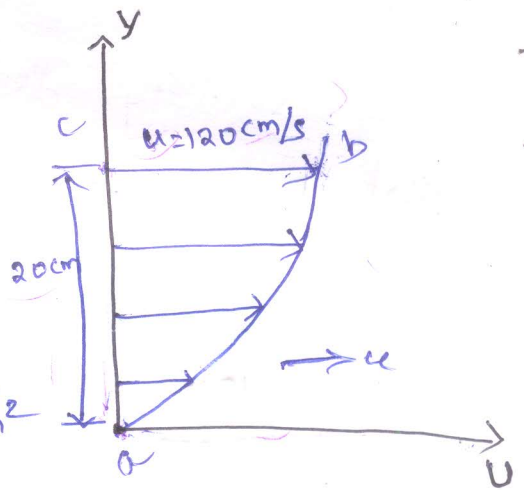
7) If the velocity profile of a fluid over a plate is parabolic with the vertex 20 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stresses at a distance of 0, 10 and 20 cm from the plate, if the viscosity of fluid is 8.5 poise.

Given :

Distance of Vertices from plate $] = 20 \text{ cm}$

Velocity at vertex B = 120 cm/sec

viscosity $\mu = 8.5 \text{ poise} = 0.85 \frac{\text{N s}}{\text{m}^2}$



Sol:

The velocity profile of given parabolic and equation of velocity profile is

$$u = ay^2 + by + c \rightarrow (v)$$

where a , b and c are constants. Their values are determined from boundary conditions are:

- a) at $y=0, u=0$
b) at $y=20\text{ cm}, u=120\text{ cm/sec}$
c) at $y=20\text{ cm}, \frac{du}{dy}=0$

Substituting boundary conditions (a) in eq (1), we get

$$C=O$$

Boundary condition (b) on substitution in (v) gives

$$120 = a(20)^2 + b(20) = 400a + 20b \rightarrow (2)$$

Boundary condition c) on substitution in (V) gives

$$\frac{du}{dy} = 2ay + b$$

$$0 = 2a(20) + b$$

$$0 = 40a + b \rightarrow 3$$

$$d = ay^2 + by + c$$

$$\frac{dy}{dx} = 2xy + b$$

Solve ② + ③

$$400a + 20b = 120 \quad \text{--- ②}$$

$$40a + b = 0 \quad \text{--- ③}$$

 $\Rightarrow$

$$400a + 20b = 120$$

$$400a + 10b = 0$$

 $(-)$ $(-)$ $(-)$

$$10b = 120 \rightarrow \boxed{b = 12}$$

③ $\times 10$ Sub $b = 12$ in ③

$$40a + 12 = 0$$

$$a = \frac{-12}{40} \Rightarrow \boxed{a = -0.3}$$

Sub a, b & c values in eq ①

$$u = -0.3x^2 + 12y \Rightarrow \frac{du}{dy} = -0.3 \times 2x + 12$$

$$= -0.6x + 12$$

$$u = -0.3x^2 + 12 \quad \swarrow \quad \searrow$$

Velocity Gradient:

$$\left(\frac{du}{dy}\right)_{y=0}^{cm/s} = (-0.6 \times 0) + 12 = 12 \text{ /s}$$

$$\left(\frac{du}{dy}\right)_{y=10} = (-0.6 \times 10) + 12 = 6 \text{ /s}$$

$$\left(\frac{du}{dy}\right)_{y=20} = (-0.6 \times 20) + 12 = 0 \text{ /s}$$

Shear Stress $\tau = \mu \frac{du}{dy}$

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12$$

$$= 10.2 \text{ N/m}^2$$

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6$$

$$= 5.1 \text{ N/m}^2$$

$$\tau = \mu \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0$$

$$= 0$$

Bulk Modulus: (K)

Ratio of change in pressure to the rate of change of volume due to the change in pressure.

$$K = - \frac{\text{change in pressure}}{\text{change in volume per original volume}} = - \frac{dp}{\left(\frac{dv}{v}\right)} \quad \text{or} \quad - \frac{dp}{\left(\frac{dl}{l}\right)}$$

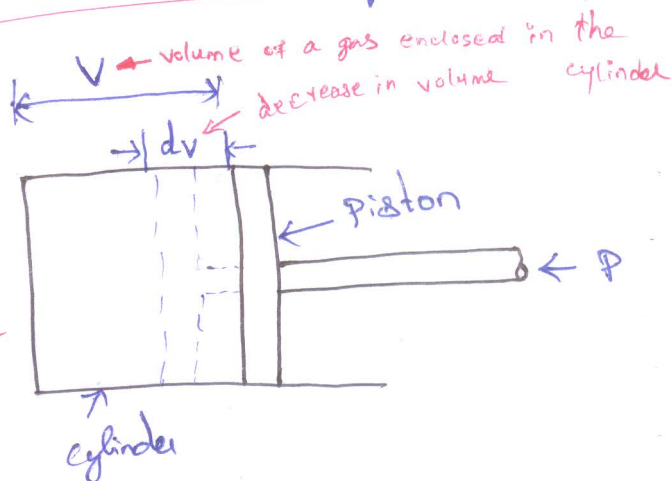
volumetric strain $\rightarrow -\frac{dv}{v}$

-ve sign [Volume decreases with increase of Pressure]

Compressibility:

Reciprocal of bulk modulus

$$\text{Compressibility} = \frac{1}{K}$$



1) what is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m^3 at 80 N/cm^2 pressure to a volume of 0.0124 m^3 at 150 N/cm^2 pressure. & also find compressibility

Given:

$$V_1 = 0.0125 \text{ m}^3$$

$$V_2 = 0.0124 \text{ m}^3$$

Sol:

$$\text{change in volume } dv = 0.0125 - 0.0124 = 0.0001 \text{ m}^3$$

$$P_1 = 80 \text{ N/cm}^2 = 80 \times 10^4 \text{ N/m}^2 \quad P_2 = 150 \text{ N/cm}^2 = 150 \times 10^4 \text{ N/m}^2$$

$$10^3 \times 10000 = 10^7$$

$$C = \frac{1}{K} = \frac{1}{10000} = 10^{-4}$$

$$1 \text{ mm}^2 = 10^{-6}$$

$$\frac{\text{cm}}{100} = \frac{\text{m}}{1000} \Rightarrow 1 \text{ cm} = 10^{-2} \text{ m}$$

$$1000 = 10^3$$

$$\text{change in pressure} = 150 - 80 = 70 \text{ N/cm}^2$$

$$= 70 \times 10^4 \text{ N/m}^2$$

$$-\frac{dv}{v} = \frac{0.0001}{0.0125} = 8 \times 10^{-3}$$

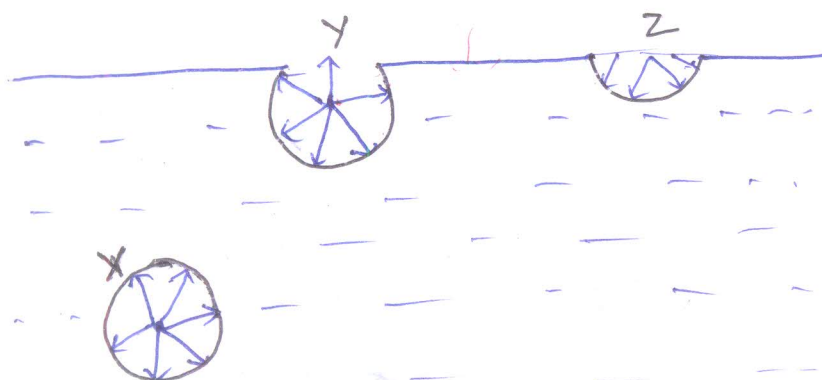
$$K = \frac{dp}{-\left(\frac{dv}{v}\right)} = \frac{70 \times 10^4}{\left(\frac{0.0001}{0.0125}\right)} = 8.75 \times 10^7 \text{ N/m}^2$$

$$\text{compressibility} = \frac{1}{K} = \frac{1}{8.75 \times 10^7} = 1.1428 \times 10^{-8} \text{ m}^2/\text{N}$$

Surface Tension: (σ)

Surface tension is defined as the tensile force required to keep unit length of the surface film in equilibrium.

✓ Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or ~~on the surface~~ ^{when} on the surface between two immiscible liquids. ~~such that the contact surface~~ behaves like a membrane under tension.



X → Force acting in all directions.

Y → near free surface.
↑ force ↓ downward force
Net resultant force in ↓ downward direction.

Z → It has net downward force.

examples: Formation of water bubbles

✓ " of rain droplets

✓ Breakup of liquid jets

1) Surface tension in liquid droplet:



water droplet



Pressure forces



Surface tension.

✓ σ = Surface tension of the liquid

✓ P = Pressure inside the droplet above outside pressure

✓ $d \rightarrow$ Diameter of the droplet

$$\text{Pressure force} = P \times \frac{\pi}{4} d^2$$

$$\text{Surface tension} = \sigma \times \pi d$$

These two forces are equal and opposite under equilibrium conditions.

$$P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$P = \frac{\sigma \times d \times 4}{d \times d} = \frac{4\sigma}{d}$$

$$P = \frac{4\sigma}{d}$$

$$d \uparrow \rightarrow P \downarrow$$

2) Surface tension on a hollow bubble:

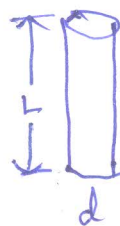
$$P \times \frac{4\sigma}{d} \times d^2 = 2(\sigma \pi d)$$

$$P = \frac{8\sigma}{d}$$

Soap bubble, bubble

3) Surface tension on a liquid jet:

$$P = \frac{2\sigma}{d}$$



24 Pg 85 Prob 1.25
 1) The surface tension of water in contact with air at 20°C is 0.0725 N/m . The pressure inside a droplet of water is to be 0.02 N/cm^2 greater ($>$) than the outside pressure. Calculate the diameter of the ~~droplet~~ droplet of water.

Given:

Surface tension $\sigma = 0.0725 \text{ N/m}$

Pressure intensity $P = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \text{ N/m}^2 = 200$

To find:

$d = ?$

Sol:

$$P = \frac{4\sigma}{d} \Rightarrow \frac{4 \times 0.0725}{d} = 0.02 \times 10^4$$

$$d = 0.00145 \text{ m}$$

✓ Vapour Pressure:

Vapour of the liquid exerts a partial Pressure on the surface of the liquid. This partial pressure is known as Vapour Pressure of the liquid.

Capillarity:

Capillarity is a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid.

This phenomenon is due to combined effect of cohesion and adhesion of liquid Particle.

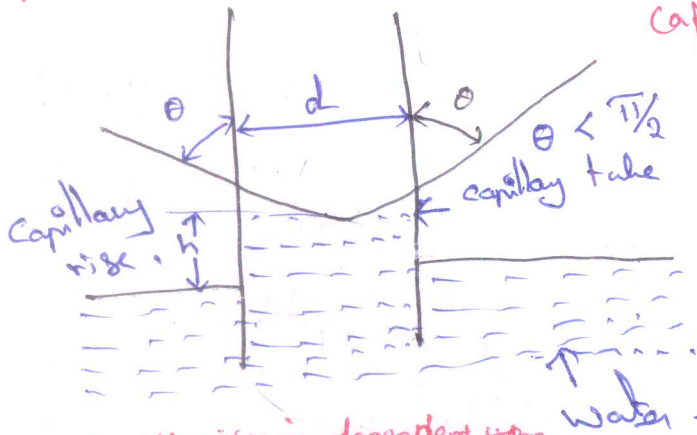
Unit
[cm (or) mm]

[illegible]

Cohesion: due to force of attraction between molecules of same liquid.

Adhesion: Defined as the force of attraction between the molecules of liquid and molecules of the solid boundary surface. $w = \rho g$

(attraction b/w two diff liquid)



Capillary
rise

$$h = \frac{40 \cos \theta}{w d}$$

$w = 99$

for water $\theta = 0$ $\theta = 0^\circ$

$$h = \frac{45 \cos 0^\circ}{\omega d} \Rightarrow \frac{45}{\omega d}$$

$\theta = 130^\circ \Rightarrow$ for mercury

capillarity is dependent upon

- dia of tube (d)
- spe weight of liquid (w)
- surface tension of liquid (σ)

last value is negative, hence h is also negative. $\therefore$ the capillary depression is caused.

- i) Calculate the capillary rise in a glass tube of 4mm diameter when immersed in i) water
 ii) Mercury, Assume $\sigma_{\text{water}} = 0.075 \text{ N/m}$ & $\sigma_{\text{m}} = 0.45 \text{ N/m}$

Given:

Diameter of glass tube, $d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$

$\sigma_{\text{water}} = 0.075 \text{ N/m}$ $\sigma_{\text{mercury}} = 0.45 \text{ N/m}$

Sol:

1. Capillary rise of water

$$h = \frac{4\sigma \cos \theta}{\rho g d} = \frac{4 \times 0.075}{9810 \times 0.004}$$

sp. weight
 $w = \rho g = 1000 \times 9.81 = 9810 \text{ N/m}^3$

$$h = 0.0076 \text{ m}$$

sp. gr. of mercury = 13.6

2. Capillary rise for mercury

$$h = \frac{4\sigma \cos \theta}{\rho g d} = \frac{4 \times 0.45 \times (-0.766)}{133416 \times 0.004}$$

$$h = - \text{ } \times 10^{-3} \text{ m}$$

capillary depression.

for mercury $\theta = 140^\circ$
 $\cos \theta = -0.766$

sp. gr. = density of given liquid / density of water
 $\rho_{\text{mercury}} = \text{sp. gr.} \times \rho_{\text{water}}$
 $\rho_{\text{mercury}} = 13.6 \times 1000$

$\theta = 140^\circ$
 $\cos \theta =$
 $w = 13.6 \times 9810$
 $= 133416 \text{ N/m}^3$
 $= 13.6 \times 1000$
 $\rho = \text{sp. gr.} \times 1000$
 $= 13.6 \times 1000$
 $= 13600 \text{ kg/m}^3$
 $w = \rho g$
 $= 13600 \times 9.81$
 $= 133416 \text{ N/m}^3$

depression

2nd Unit

Types of fluid flow:

at a point

Veloc, Press, ρ does not change with time.

change with time.

1. Steady & unsteady flow

2. Uniform & non uniform flow

3. Laminar & turbulent flows **Turbulent**

4. Compressible & incompressible flows

5. Rotational & irrotational flows.

6. One, two & three dimensional flows

2. Uniform flow: **non-uniform flow**

It is a type of flow in which the velocity at any given time does not change with length or direction of the flow.

respect to space (or)

3. Laminar:

In this fluid particles move along well-defined paths or stream line and the stream lines are straight and parallel.

This also called stream line flow or viscous flow.

Turbulent: **Turbulent flow**

In this type of flow, fluid particles move in a zig-zag way.

Due to zig-zag way, the eddies formation takes responsible for high energy loss.

$Re < 2000 \rightarrow$ laminar flow

$Re > 4000 \rightarrow$ Turbulent flow

$Re > 2000 \& < 4000 \rightarrow$ Laminar or Turbulent

4. Compressible: density changes from point to point

ρ is ~~can~~ not constant $\rho \neq \text{constant}$

Incompressible: ρ is constant

$\rho \neq \text{constant}$ | gases are compressible.

5. Rotational flow:

Fluid particles while flowing along streamlines, also rotate about their own axis.

Irrotational flow:

Fluid particles while flowing along streamlines, do not rotate about their own axis.

6. One dimensional: let u, v, w are velocity components in x, y and z directions.



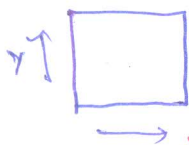
one space only (or)

other two faces negligible

$$u = f(x) \quad v = 0 \quad w = 0$$

velocity changes in only one direction (or), other two direction negligible

Two dimensional:



$$u = f_1(x, y) \quad v = f_2(x, y) \quad w = 0$$

Three dimensional:



$$u = f_1(x, y, z) \quad v = f_2(x, y, z) \quad w = f_3(x, y, z)$$

dynamic viscosity

(3)

$$\frac{1}{10} \frac{\text{N-s}}{\text{m}^2} = \text{poise}$$

$$\text{poise} = 100 \text{ centipoise}$$

$$\frac{1}{10} \frac{\text{N-s}}{\text{m}^2} = 100 \text{ cp}$$

$$\frac{1}{1000} \frac{\text{N-s}}{\text{m}^2} = \text{cp}$$

$$\checkmark \quad \boxed{\text{cp} = 10^{-3} \frac{\text{N-s}}{\text{m}^2}}$$

kinematic viscosity

$$\checkmark \quad \frac{1}{10^4} \frac{\text{m}^2}{\text{s}} = \text{stoke}$$

$$\text{stoke} = 100 \text{ centistoke}$$

$$\frac{1}{10^4} \frac{\text{m}^2}{\text{s}} = 100 \text{ cs}$$

$$\frac{1}{10^4 \times 100} \frac{\text{m}^2}{\text{s}} = \text{cs}$$

$$\text{cs} = \frac{1}{10^6} \frac{\text{m}^2}{\text{s}}$$

$$\checkmark \quad \boxed{\text{cs} = 10^{-6} \frac{\text{m}^2}{\text{s}}}$$

Flow characteristics:

Discharge (or) Rate of flow: (Q)

It is defined as the volume of a fluid flowing across the section of a pipe per second.

$$Q = A \times V \quad \text{m}^3/\text{s}$$

$$\text{m}^2 \times \text{m}/\text{s} \Rightarrow \text{m}^3/\text{s}$$

A = Cross-sectional area of pipe

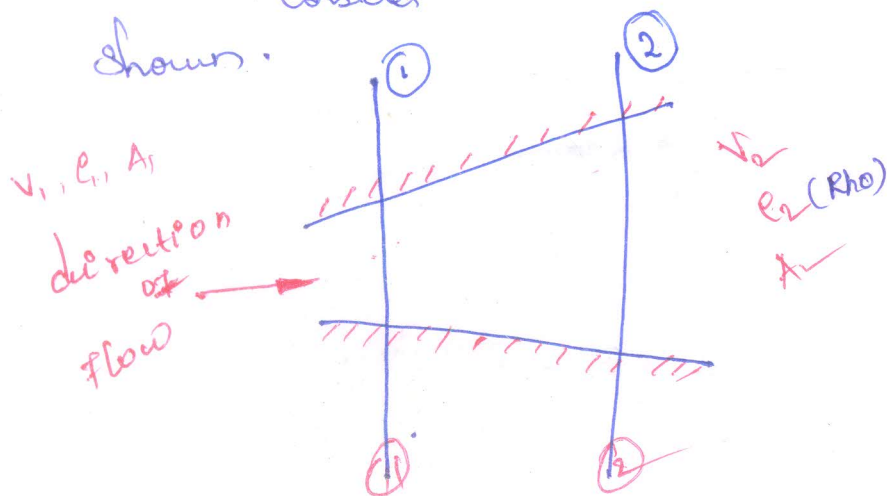
V = Average velocity of fluid across the section.

Continuity equation:

The continuity equation is based on the Principle of conservation of mass is called Continuity equation.

It states that the mass of fluid flowing through the pipe at all cross section remains constant, if there is no fluid is added or removed from the pipe.

Consider two cross-sections of a pipe as shown.



Let V_1 = Avg Velocity at cross section 1-1,

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1.

V_2, ρ_2, A_2 , are corresponding values at section 2-2.

According to law of Conservation of mass.

Rate of flow @ 1-1 = Rate of flow at 2-2

✓ $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

If the fluid is incompressible, ^{water} then $\rho_1 = \rho_2$

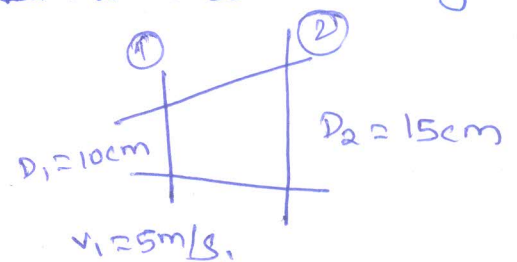
$$A_1 V_1 = A_2 V_2$$

$A = \frac{\pi D^2}{4}$

1. The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5m/s. Determine also velocity at section 2.

Given:

$D_1 = 10\text{cm} = 0.1\text{m}$ $D_2 = 15\text{cm} = 0.15\text{m}$
 $V_1 = 5\text{m/s}$



Sol: $A_1 = \frac{\pi}{4} \times D_1^2 = 0.007854\text{m}^2$
 $A_2 = \frac{\pi}{4} \times D_2^2 = 0.01767\text{m}^2$

$Q = A_1 V_1 = 0.007854 \times 5 = 0.03927\text{m}^3/\text{s}$

$A_1 V_1 = A_2 V_2$

$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.007854 \times 5}{0.01767} = 2.22\text{m/s}$

$V = \frac{\text{m}}{\text{s}} \rightarrow \text{m}^2/\text{s}$
 $A = \text{m}^2$

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100% Bansal

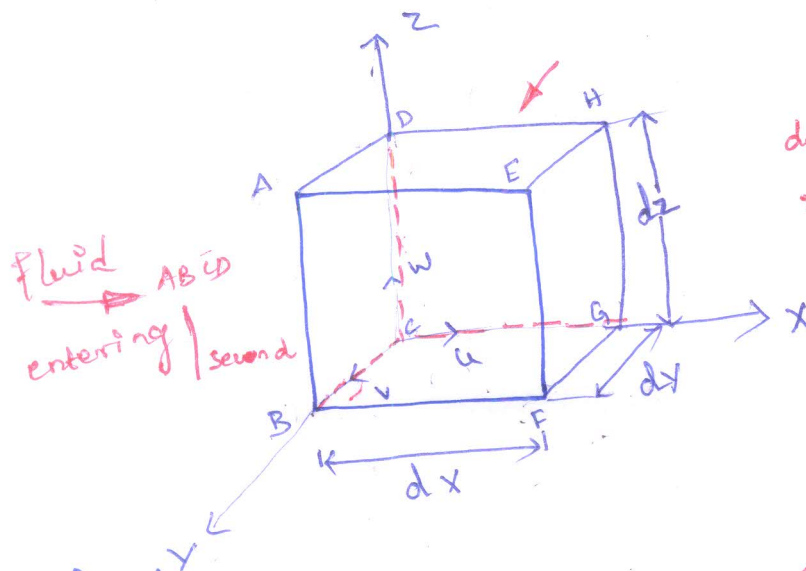
163 Prob 5.2

Continuity equation in differential form (or)

Three dimensional form:

Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z .

Let u, v and w are the inlet velocity components in x, y and z directions respectively.



$u, v, w \rightarrow$ inlet velocity

$dx, dy, dz \rightarrow$ fluid element or lengths

$x, y, z \rightarrow$ direction

x-direction:

$$\left. \begin{array}{l} \text{Mass of fluid entering} \\ \text{the face ABCD per second} \end{array} \right\} = \rho \times \text{Area of ABCD} \times \text{velocity in x-direction.}$$

$$= \rho \times (dy \times dz) \times u$$

$$\left. \begin{array}{l} \text{Then mass of fluid} \\ \text{leaving the face EFGH / second} \end{array} \right\} = \rho u dy dz + \frac{\partial}{\partial x} (\rho u dy dz) dx$$

$$\text{Gain of mass in x-direction} = \text{Mass through ABCD} - \text{Mass through EFGH}$$

$$= \rho u dy dz - \left[\rho u dy dz + \frac{\partial}{\partial x} (\rho u dy dz) dx \right]$$

$$= - \frac{\partial}{\partial x} (\rho u) dx dy dz.$$

y-direction: (B)

Mass of fluid entering the face DCGH per sec = $\rho \times \text{Area of DCGH} \times \text{velocity in y-direction}$

$$= \rho \times (dx dz) v$$

$$= \rho v \times (dx dz)$$

Mass of fluid leaving ABFE

$$= \rho v dx dz + \frac{\partial}{\partial y} (\rho v dx dz) dy$$

Net gain of mass in y-direction

$$= -\frac{\partial}{\partial y} (\rho v) dx dy dz$$

z-direction:

Mass of fluid entering the face BCGF per sec = $\rho w dx dy$

Mass of fluid leaving the face ~~BCGF~~ / s

$$= \rho w dx dy + \frac{\partial}{\partial z} (\rho w dx dy) dz$$

ADEH / s

Gain of mass in z-direction

$$= -\frac{\partial}{\partial z} (\rho w) dx dy dz$$

Net gain of masses

$$= - \left[\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] dx dy dz \rightarrow (1)$$

Since Mass is neither created nor destroyed in the fluid element, ~~must~~ the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.

(C)

mass of fluid }
in the element is } $\rho \cdot dx \cdot dy \cdot dz$

Rate of increase }
with time is } $\frac{\partial}{\partial t} (\rho \cdot dx \cdot dy \cdot dz)$ (or) $\frac{\partial \rho}{\partial t} (dx dy dz)$ $\rightarrow$ (2)

Equating (1) $\rightarrow$ (2)

$$-\left[\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] dx dy dz = \frac{\partial \rho}{\partial t} \cdot dx dy dz$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0 \rightarrow (3)$$

Eq (3) is the continuity eqn in cartesian coordinates in its most general form.

This eqn is applicable to

- i) Steady and unsteady flow
- ii) Uniform and non-uniform flow
- iii) Compressible and incompressible fluids.

For steady flow, $\frac{\partial \rho}{\partial t} = 0$

$$(3) \Rightarrow \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0$$

If the fluid is incompressible, $\rho = c$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \rightarrow \text{Continuity eq in 3 dimensions.}$$

For two dimensional flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Formulas:

$$A = \frac{\pi}{4} d^2$$

$$Q = A_1 V_1 = A_2 V_2 \quad \text{continuity eqn}$$

Bernoulli's eqn:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 \quad \text{or} \quad \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 + H_p = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2$$

$$\gamma = \rho \times g$$

$$\gamma = 9.81 \times 1000 \quad \text{for water}$$

Power (P)

exg

$$\left\{ \begin{array}{l} \text{Energy @ Point A} = \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 \\ \text{@ point B} = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 \end{array} \right.$$

Power delivered by pump of water: $P = \gamma \times Q \times H_p$

$$\text{Power required to run the pump (P)} = \frac{\gamma \times Q \times H_p}{\eta_p}$$

$$1 \text{ Pa} = \text{N/m}^2$$



energy added by pump

Prob 1: 1.115 (P1)

Prob 2

Prob 6: 1.120

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + H_p = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + (Z_2 - Z_1)$$

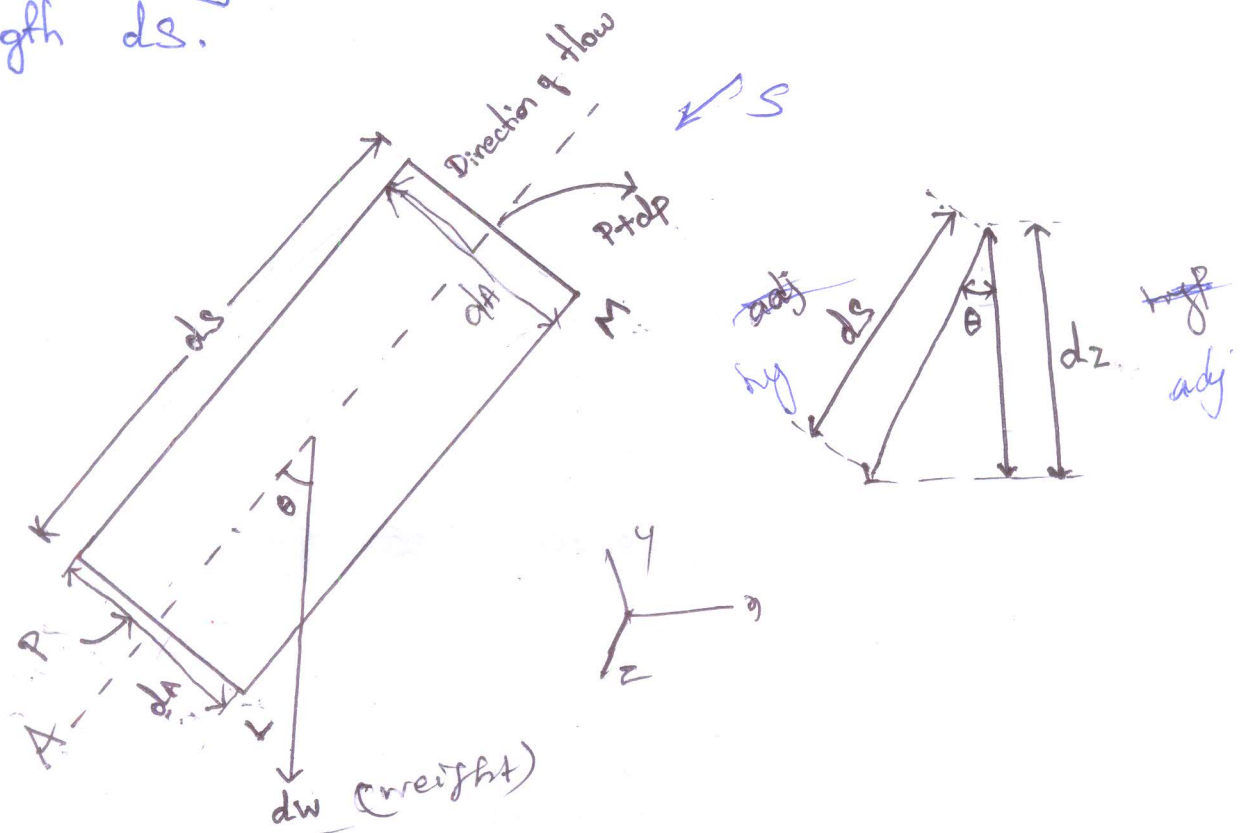
$$h = \left(\frac{P_1}{\gamma} + Z_1 \right) - \left(\frac{P_2}{\gamma} + Z_2 \right)$$

$$\begin{aligned} 0.275 - 0.08168 + 0.049 \text{ vs} \\ 0.2143 = 0.049 \text{ vs} \\ \hline 0.049 \\ = 4.322 \end{aligned}$$

Application of Energy Equation:

Euler's equation:

Consider a steady flow of an ideal fluid along a streamline with a small element of the flowing fluid LM of cross section dA and length ds .



Let, P = Pressure on the element at A.

$P+dp$ = Pressure on the element at M and

v = Velocity of fluid element.

We know that the net force acting on the fluid element in the direction of flow.

$$= P \cdot dA - (P+dp) dA = -dp dA \rightarrow \textcircled{1}$$

$$= \cancel{P dA} - \cancel{P dA} - dp dA$$

$$= -dp dA \rightarrow \textcircled{1} \text{ Force}$$

(b)

We also know that the weight of the fluid element is

$$dw = \rho g \cdot dA \cdot ds$$

Specific weight

$$\text{Spec. weight} = \frac{W}{V}$$

$$\rho = \frac{m}{V}$$

$$\rho g = \frac{m \cdot g}{V} = \frac{W}{V} \text{ --- volume.}$$

$$\rho g \times V = W$$

Component of weight of the fluid element in the direction of flow.

$$= -\rho g \cdot dA \cdot ds \cdot \cos \theta = -\rho g \cdot dA \cdot ds \cdot \left(\frac{dz}{ds} \right)$$

$$= -\rho g \cdot dA \cdot dz \rightarrow (2)$$

The resultant force on the fluid element in the direction of s

$$= -dp \cdot dA - \rho g dA dz \rightarrow (3)$$

$$\text{Mass of the fluid element} = \rho \cdot dA \cdot ds \rightarrow (4)$$

The acceleration of the fluid element

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v \cdot \frac{dv}{ds} \rightarrow (5)$$

According to the Newton's second law of motion,

$$\text{Force} = \text{Mass} \times \text{Acceleration.}$$

(3)

(4)

(5)

$$(-dp \cdot dA) - (\rho g dA dz) = \rho dA ds \times v \times \frac{dv}{ds}$$

$$\rho dA \times \rho dA$$

$$-dp - g dz = v \frac{dv}{ds} \Rightarrow -\frac{dp}{\rho} - g dz = v dv$$

÷ both sides by $\rho \cdot dA$

(C)

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$$-\frac{dp}{\rho} - g dz = v \times dv$$

$$\boxed{\frac{dp}{\rho} + v dv + g dz = 0}$$

→ (6)

Euler's equation of Motion.

Bernoulli's Equation from Euler's Equation:

∫ eq (6)

$$\frac{1}{\rho} \int dp + g \int dz + \int v dv = C \quad \int dx = x$$

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = C$$

÷ g

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = C$$

eq

$$\frac{P}{w} + \frac{v^2}{2g} + z = C$$

$$\frac{P_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{v_2^2}{2g} + z_2$$

Bernoulli's equation:

Bernoulli's equation relates velocity, Pressure and equation changes of a fluid in motion.

Bernoulli's equation: (d)

"In an ideal, incompressible fluid when the flow is steady and continuous, the sum of Pressure energy, kinetic energy and potential energy is constant along a stream line"

$$\frac{P}{w} + \frac{v^2}{2g} + z = \text{Constant}$$

where

$\frac{P}{w} \Rightarrow$ Pressure Energy

$\frac{v^2}{2g} \Rightarrow$ Kinetic Energy

$z \Rightarrow$ Datum Energy

Potential Energy:

Energy which results from position or configuration.

$\rightarrow$ the path of a particle in a fluid relative to a solid body that is moving in smooth flow

Assumptions:

1. The fluid is ideal i.e (viscosity is zero)
2. The flow is steady
3. The flow is incompressible.
4. The flow is irrotational.

Applications of Bernoulli's Equation:

- $\rightarrow$ Venturimeter
- $\rightarrow$ Orificemeter
- $\rightarrow$ Pitot tube

$w = \rho g = 5 \times 1000 \times 9.81$
 $= 49050$

Bernoulli's

pg no
(1.16)
problems

eg. 2.8

- 1) The water is flowing through a pipe having diameter of 20cm and 10cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm^2 , find the intensity of pressure at section 2.

Given:

$$D_1 = 20\text{cm} = 0.2\text{m} \quad D_2 = 10\text{cm} = 0.1\text{m}$$

$$P_1 = 39.24 \text{ N/cm}^2$$

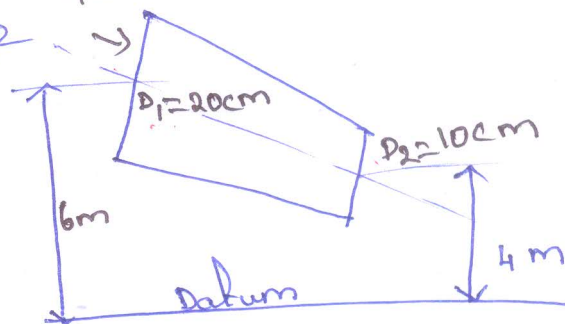
$$P_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6\text{m} \quad z_2 = 4\text{m}$$

$$Q = 35 \text{ lit/s} = 35 \times 10^{-3} \text{ m}^3/\text{s}$$

To find:

$$P_2 = ?$$



Sol:

$$A_1 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 \text{ m}^2 \quad A_2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785 \text{ m}^2$$

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 \text{ m/s} \quad V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at ① & ② sections

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{4.456^2}{2 \times 9.81} + 4.0$$

$$P_2 = 4.02 \times 10^5 \text{ N/m}^2$$

Applications of Bernoulli's Equation:

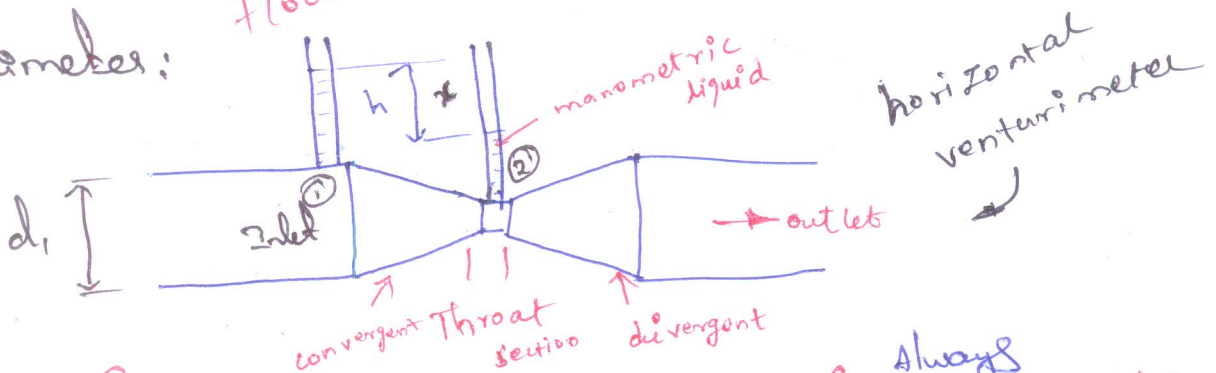
1. Venturimeter
2. Orifice meter
3. Pitot - tube.

By reducing the cross sectional area, a pressure difference is created.

↓
due to maximum discharge.

Flow measurement device.

1. Venturimeter:



$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

C_d Always < 1 value.
↓
discharge
Co-efficient of venturimeter

Value of 'h' given by differential U-tube manometer:

~~case~~ i) Horizontal Venturimeter: $S_h > S_o$ case (i)
 $S_h \rightarrow$ sp. gravity of heavier liquid (13.6)
 $S_o \rightarrow$ " liquid in pipe (0.8 or 1)
 $x \rightarrow$ Difference of heavier liquid column in U tube
 S_h (or) S_m

~~case~~ ii) Inclined venturimeter:

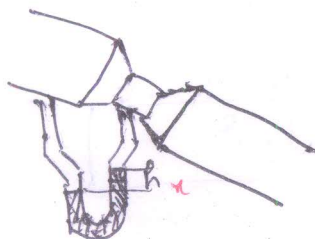
$$h = \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right) = x \left[\frac{S_h}{S_o} - 1 \right]$$

↑
 $w = \rho g$

case (ii)

if $\left(1 - \frac{S_h}{S_o} \right) S_h > S_m$

$S_h < S_o$



$h = \left(\frac{P_1}{\rho g} + z_1 \right) - \left(\frac{P_2}{\rho g} + z_2 \right)$
↓
Head difference due to pressure difference

- 1) A horizontal venturimeter with inlet and throat diameters 30cm and 15cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20cm of mercury. Determine the rate of flow. Take $C_d = 0.98$. [Pg. 268, Prob 6.11]

Given:

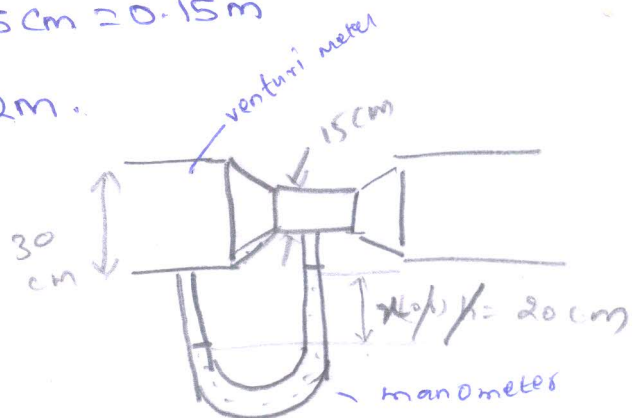
$$d_1 = 30\text{cm} = 0.3\text{m} \quad d_2 = 15\text{cm} = 0.15\text{m}$$

$$C_d = 0.98 \quad x = 20\text{cm} = 0.2\text{m}$$

To find:

$$Q = ?$$

$$\frac{20}{100}$$



Sol:

$$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 0.3^2 = 0.0706\text{m}^2$$

$$a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times 0.15^2 = 0.0176\text{m}^2$$

$$h = x \left[\frac{S_h}{S_o} - 1 \right]$$

$$\rightarrow \begin{matrix} S_h = 13.6 \\ S_o = 1 \end{matrix}$$

$$\begin{matrix} S_o = 0.8 \text{ (oil)} \\ \text{sp. gr. of oil} = 0.8 \\ \text{sp. gr. of mercury} \end{matrix}$$

$$S_h = 13.6$$

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.1257 \text{ m}^3/\text{s} \times 1000$$

$$= 125.7 \text{ lit/s}$$

Inclined Venturimeter:

- 1) A 30 cm x 15 cm Venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives reading of 20 cm. Find the discharge. Take $C_d = 0.98$.

Given: (pg. 271 - Prob 6.15)

$$d_1 = 0.3 \text{ m}$$

$$30 \text{ cm} = \frac{30}{100} = 0.3 \text{ m}$$

$$d_2 = 0.15 \text{ m}$$

$$x = 0.2 \text{ m}$$

$$C_d = 0.98$$

Assume: $S_h = 13.6$

$$S_o = 1 \text{ (or) } 0.8$$

To find:

$$Q = ?$$

Sol:

$$a_1 = \frac{\pi}{4} \times (0.3)^2 = 0.0706 \text{ m}^2$$

$$a_2 = \frac{\pi}{4} \times (0.15)^2 = 0.0176 \text{ m}^2$$

$$h = x \left[\frac{S_h}{S_o} - 1 \right] = 2.52 \text{ m}$$

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.1257 \text{ m}^3/\text{s}$$

$$= 125.756 \text{ lit/s}$$

co-efficient of velocity

$$C_v = \frac{x}{\sqrt{4gh}} \text{ (or) } \frac{C_d}{C_c}$$

co-efficient of discharge C_d

$$C_d = \frac{Q_{act}}{Q_{th}} \text{ (no unit)}$$

co-efficient of contraction

$$C_c = \frac{C_d}{C_v} \text{ (or) } \frac{a_2}{a_o}$$

$a_2 \rightarrow$ area of vena-contracta

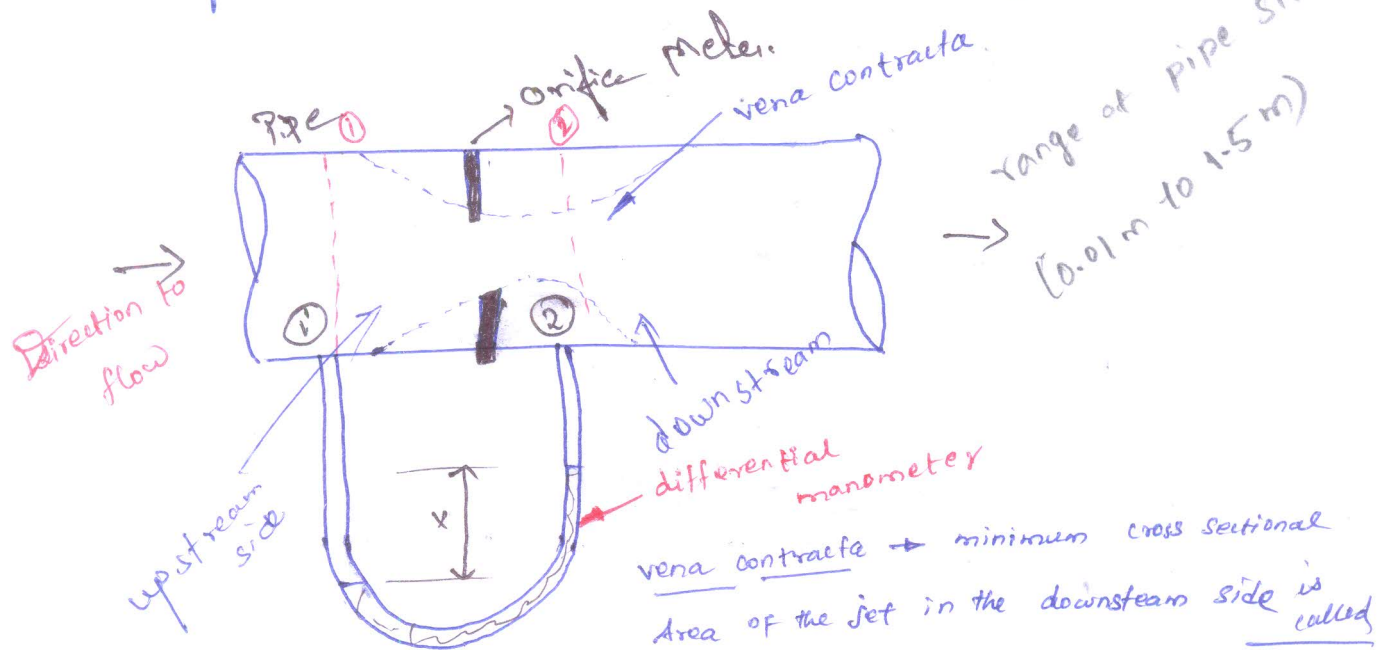
$a_o \rightarrow$ Area of orifice

$$Q = \text{Volume} / \text{time}$$

Orifice Meter: (278 Pg)

1. It is cheaper

2. Orifice dia 0.5 times the dia of Pipe



$$Q = \frac{C_d a_o a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_o^2}}$$

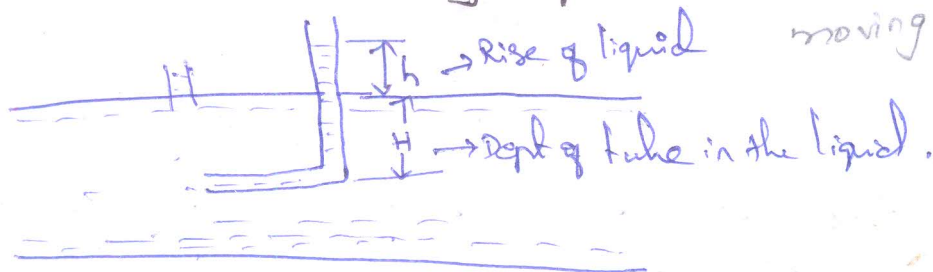
$Q \rightarrow$ discharge

$C_d \rightarrow$ coefficient of discharge

$a_o \rightarrow$ Area of orifice

$a_1 \rightarrow$ theoretical velocity

Pitot tube: [velocity of flow] find the discharge of a moving liquid



$$V = C_p \sqrt{2gh}$$

C_p - coefficient of Pitot tube

$$C_c = \frac{\text{Area at vena contracta}}{\text{Area of an orifice}}$$

$$= \frac{a_2}{a_o}$$

$C_c \rightarrow$ coefficient of contraction

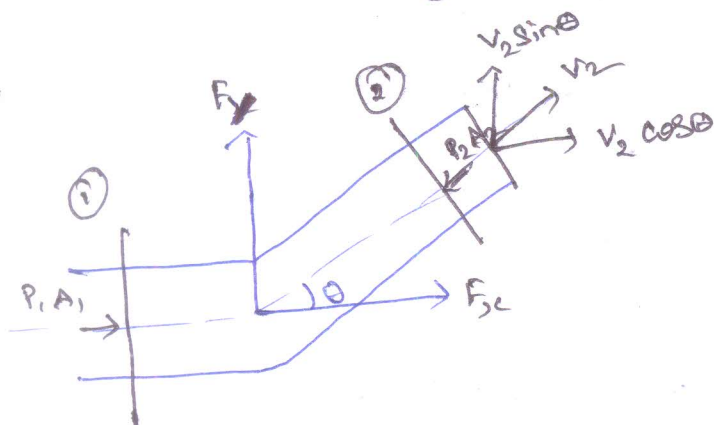
for Orifice meter

$$h = x \left(\frac{S_h}{S_o} - 1 \right)$$

$$F = m \times a = m \times \frac{dv}{dt} = \frac{d(mv)}{dt}$$

The Momentum ^{$m \times v$} equation:

force exerted by a ^{flowing} fluid on a pipe bend:



Net force acting on x-direction } $F_x = \rho Q (V_1 - V_2 \cos \theta) + P_1 A_1 - P_2 A_2 \cos \theta$

$$F_y = \rho Q (-V_2 \sin \theta) - P_2 A_2 \sin \theta$$

Resultant force

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$\tan \theta = \frac{F_y}{F_x}$$

application

1) flow problem analysis in turbines & centrifugal pumps.

- 1) A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600mm and 300mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm^2 and rate of flow of water is 600 lit/s .

- & angle made by resultant force.

Given:

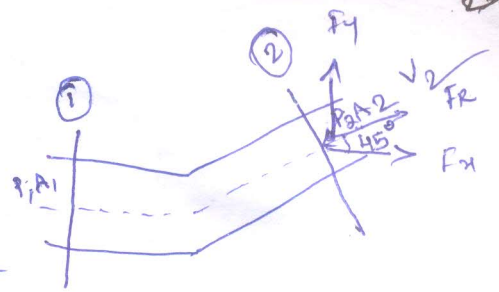
$$\theta = 45^\circ$$

$$D_1 = 600 \text{ mm} = 0.6 \text{ m}$$

$$D_2 = 300 \text{ mm} = 0.3 \text{ m}$$

$$P_1 = 8.829 \text{ N/cm}^2 = 8.829 \times 10^4 \text{ N/m}^2$$

$$Q = 600 \text{ lit/s} = 0.6 \text{ m}^3/\text{s}$$



Sol:

$$A_1 = \frac{\pi}{4} (D_1^2) = \frac{\pi}{4} (0.6^2) = 0.2827 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} (D_2^2) = \frac{\pi}{4} (0.3^2) = 0.07068 \text{ m}^2$$

$$Q = AV \quad \therefore V_1 = \frac{Q}{A_1} = \frac{0.6}{0.2827} = 2.122 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{0.0706} = 8.488 \text{ m/s}$$

Applying Bernoulli's eqn at sections (1) & (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$$\frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{P_2}{1000 \times 9.81} + \frac{8.488^2}{2 \times 9.81}$$

$$P_2 = 5.45 \times 10^4 \text{ N/m}^2$$

Forces on the bend in x and y directions are

$$F_x = \rho Q [v_1 - v_2 \cos \theta] + P_1 A_1 - P_2 A_2 \cos \theta$$

$$= 1000 \times 0.6 [2.122 - 8.488 \cos 45] + 8.829 \times 10^4 \times 0.2827 - 5.45 \times 10^4 \times 0.0706 \times \cos 45^\circ$$

$$= 19911.4 \text{ N}$$

$$F_y = \rho Q [-v_2 \sin \theta] - P_2 A_2 \sin \theta$$

$$= 1000 \times 0.6 [-8.488 \times \sin 45^\circ] - 5.45 \times 10^4 \times 0.07068 \times \sin 45^\circ$$

$$= -6322.2 \text{ N}$$

→ F_y getting in downward direction.

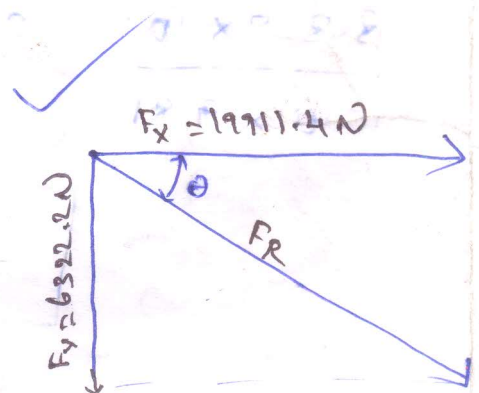
$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(19911.4)^2 + (-6322.2)^2}$$

$$= 20890.9 \text{ N}$$

The angle made by Resultant force with x-axis is

$$\tan \theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4}$$

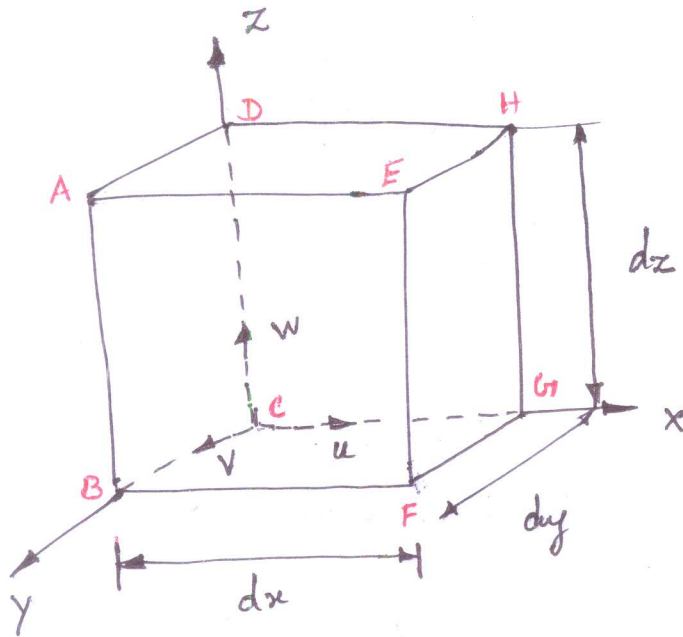
$$\theta = 17^\circ 36'$$



①

Continuity Equation in differential form (or) Three dimensional form:

Consider a fluid element of lengths dx, dy, dz , in the direction x, y , & z . Let u, v, w are the inlet velocities components in x, y, z directions respectively.



$$\dot{m} = \frac{m}{\text{area}} \times \frac{m/s}{\text{velocity}} = m^3/s$$

x-direction:

mass of fluid entering the face ABCD per second } = \rho \times \text{Area of ABCD} \times \text{velocity in x direction}

$$\rho = \frac{m \rightarrow \text{mass}}{V \rightarrow \text{volume}}$$

$$m = \rho \times V$$

$$= \rho \times (dy \times dz) \times u$$

Then mass of fluid leaving the face EFGH / sec } = \rho u dy dz + \frac{\partial}{\partial x} (\rho u dy dz) dx

Gain of mass in x-direction:

$$\text{mass through } \begin{matrix} \text{ABCD} \\ \text{EFGH} \end{matrix} - \text{mass through } \begin{matrix} \text{EFGH} \\ \text{ABCD} \end{matrix}$$

③

$$= -\rho u dy dz + \left[\rho u dy dz + \frac{\partial}{\partial x} (\rho u dy dz) dx \right]$$

$$= -\cancel{\rho u dy dz} + \cancel{\rho u dy dz} + \frac{\partial}{\partial x} (\rho u) dy dz dx$$

$$= + \frac{\partial}{\partial x} (\rho u) dx dy dz$$

y-direction:

$$\left. \begin{array}{l} \text{mass of fluid entering} \\ \text{the face DCGH / Second} \end{array} \right\} = \rho \times \text{Area of DCGH} \times \text{velocity in y direction}$$

$$= \rho \times (dx dz) \times v$$

$$= \rho v (dx dz)$$

$$\left. \begin{array}{l} \text{mass of fluid leaving} \\ \text{ABFE / sec} \end{array} \right\} = \rho v dx dz + \frac{\partial}{\partial y} (\rho v dx dz) dy$$

$$\text{ABFE} - \text{DCGH} / \rho v dx dz + \frac{\partial}{\partial y} (\rho v dx dz) dy - \rho v dx dz$$

$$\left. \begin{array}{l} \text{net gain of mass in} \\ \text{y direction} \end{array} \right\} = + \frac{\partial}{\partial y} (\rho v) dx dy dz$$

z-direction:

$$\rho = \frac{m}{V} = m \times \rho v$$

$$\left. \begin{array}{l} \text{mass of fluid entering} \\ \text{the face BCGF / sec} \end{array} \right\} = \rho w dx dy$$

$$\left. \begin{array}{l} \text{mass of fluid leaving} \\ \text{The face ADEH / sec} \end{array} \right\} = \rho w dx dy + \frac{\partial}{\partial z} (\rho w dx dy) dz$$

$$\text{ADEH} - \text{BCGF}$$

$$\left. \begin{array}{l} \text{Gain of mass in} \\ \text{z-direction} \end{array} \right\} = + \frac{\partial}{\partial z} (\rho w) dx dy dz$$

$$\text{net gain} = \left[\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] dx dy dz$$

According to the principle of conservation of mass, there is no accumulation of mass & hence the above quantity must be zero.

$$\left[\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] dx dy dz = 0$$

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0$$

This equation is the general equation of continuity in 3D which is applicable to any type of flow & for any fluid whether compressible or incompressible.

For incompressible fluids, $\rho = \text{const}$. then the above eqn reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

For two-dimensional flow, The component $w=0$ & hence, the continuity eqn becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Unit -2

Flow Through Circular Conduits

A conduit is a pipe, tube or duct that is completely filled with the fluid.

Losses ^{of Energy} in pipes:

When a fluid flowing through a pipe, certain resistance is offered to the flowing fluid it results in causing a loss of energy.

The loss of energy is classified as:

1. Major losses → due to friction
2. Minor losses → Bend in pipe, pipe fittings, sudden expansion, contraction

Head loss:

The drop in pressure, velocity and potential head between two points.

Major Losses in pipe flow:

The major loss of energy is caused by friction in pipe.

It may be computed by Darcy-Weisbach eqn (flow through pipes)

It is given as

$$\text{Head loss due to friction, } h_f = \frac{4fLV^2}{2gD}$$

$$Re = \frac{V \times d}{\nu} \quad \leftarrow \text{kinematic viscosity, } \nu = \frac{\mu}{\rho}$$

f = Darcy's coefficient of friction

$$f = \frac{16}{Re} \quad \text{for } Re < 2000 \quad (\text{Laminar flow})$$

$$f = \frac{0.079}{(Re)^{0.25}} \quad \text{Re varying from 4000 to } 10^6$$

Re - Reynolds no.
 L - length of pipe
 V - velocity of fluid
 D - Diameter of pipe

Applications of Darcy & Chezy's formula

Darcy-Weisbach eqn is used for the flow through pipes.

" " flow through open channels.

Chezy's formula

Chezy's formula: (flow through open channels)

V - velocity of flow

C - Chezy's Coefficient

$$V = C \sqrt{mi}$$

Power: (P)

$$= \frac{eg \times Q \times h_f}{1000} \text{ KW}$$

$$m = \text{Hydraulic mean depth} = \frac{\text{Area of the pipe}}{\text{Perimeter}}$$

$$= \frac{A}{P} = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4} \quad \left| \quad c = \sqrt{eg/f'} \right.$$

$$i = \text{Loss of head per unit length of pipe} = \frac{h_f}{L}$$

Minor losses in pipe flow:

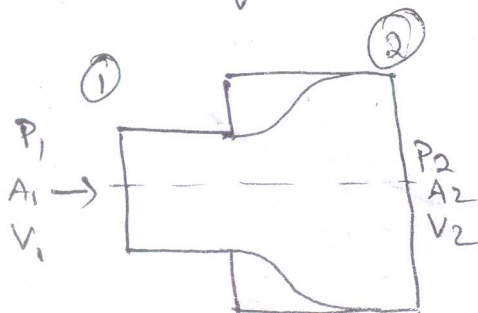
while the loss of energy caused on account of the change in velocity of flowing fluid is called minor loss of energy.

Long Pipes - These losses are small when compared to friction losses.

Short pipes - These losses are comparable with the loss of energy due to friction.

Some of the losses of energy, which may cause due to change of velocity

① Loss of energy due to sudden enlargement (he)



P_1 - Pressure at section 1

A_1 - Area of pipe at section 1

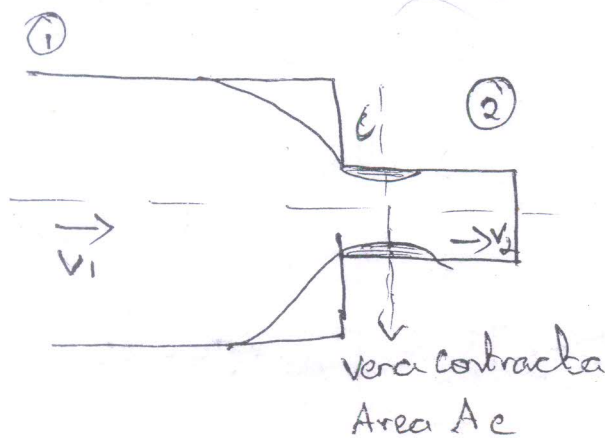
V_1 - Velocity of flow at section 1

P_2, A_2, V_2

$$h_c = \frac{(v_1 - v_2)^2}{2g} \Rightarrow \text{Loss of head due to sudden enlargement (} h_c \text{)}$$

② Loss of energy due to Sudden Contraction (h_c).

Due to sudden contraction, the streamline converges to a minimum cross section called the Vena Contracta, and then expands to fill the downstream pipe.



A_c = Area of flow at section C

v_c = Velocity of flow C

A_2 = 2

v_2 = 2

h_c = Loss of head due to sudden contraction.

Loss of head due to sudden contraction

$$h_c = \frac{v_2^2}{2g} \left[\frac{1}{C_c} - 1 \right] \quad C_c = \frac{A_c}{A_2}$$

$K \rightarrow 0.375 \text{ to } 0.5$

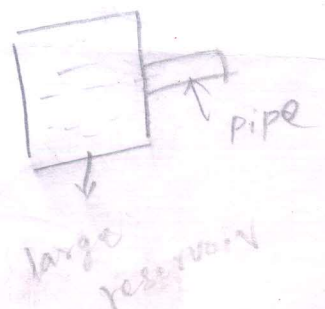
If C_c or K is not given

$$h_c = \frac{0.5 v_2^2}{2g} \quad K = \left(\frac{1}{C_c} - 1 \right)$$

③ Loss of energy at the entrance of a pipe (h_i)

Due to sudden contraction, fluid from large reservoir to pipe entrance.

Loss of head at the entrance of the pipe, $h_i = \frac{0.5 v^2}{2g}$



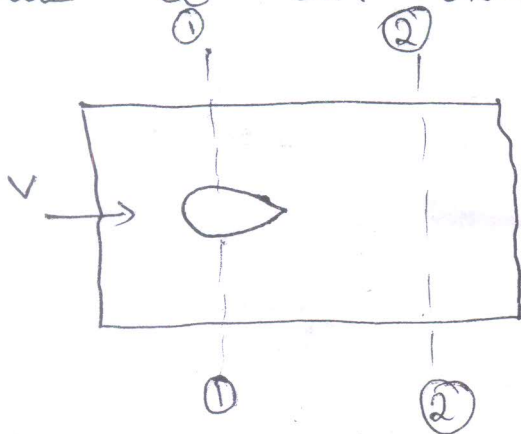
4. Loss of energy at the exit from a pipe (h_o)

Outlet of the pipe may be connected to a large reservoir.

$$h_o = \frac{V^2}{2g}$$

5. Loss of energy due to an obstruction in pipe (h_{obs})

Reduction in cross-sectional area of the pipe due to an obstruction.



V_c - velocity of liquid at Vena contracta

A_c - Area of cross section at Vena contracta.

Vena contracta formed beyond

$$h_{obs} = \frac{V^2}{2g} \left(\frac{A}{C_c(A-a)} - 1 \right)^2$$

$$A_c = C_c(A-a)$$

$$C_c = \frac{\text{Area of Vena contracta}}{\text{Area of obstruction}} = \frac{A_c}{(A-a)}$$

Area of Vena contracta

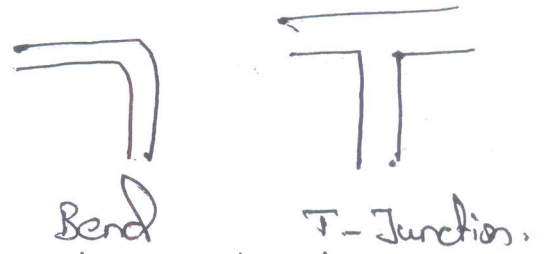
obstruction of max area.
A - pipe cross-sectional area

6. Loss of energy due to gradual contraction or enlargement.

$$h_L = K \frac{(V_1 - V_2)^2}{2g}$$

K - coefficient which depends on the angle of convergence or divergence.

⑦ Loss of energy due to bend in Pipe (h_b)
 formation of eddies resulting in the dissipation of energy.



$$h_b = \frac{K V^2}{2g}$$

K - Co-efficient of bend
 K depends on:
 i) Angle of bend
 ii) Radius of curvature of bend
 iii) Diameter of pipe

⑧ Loss of energy in various pipe fittings (h_v)

Pipe fittings such as valves, couplings etc.
Valves

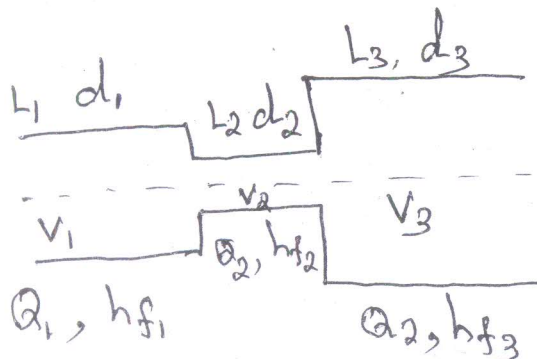
$$h_v = \frac{K V^2}{2g}$$

K - Coefficient, which depends on the type of pipe fitting.

Flow through pipes in series and parallel:

Pipes in series or compound pipe:

It is defined as the pipes of different diameters and lengths which are connected with one another to form a single pipeline.



The total loss of head through the entire system is the sum of the losses in all individual pipes.

$$H = \overset{h_{i-\text{entrance}}}{\frac{0.5V_1^2}{2g}} + \frac{4f_1 L_1 V_1^2}{2g D_1} + \underset{\substack{\downarrow \\ h_c \rightarrow \text{contraction-} \\ \text{entrance}}}{\frac{0.5V_2^2}{2g}} + \frac{4f_2 L_2 V_2^2}{2g D_2} + \frac{(V_2 - V_3)^2}{2g} + \underset{\substack{\downarrow \\ h_e \text{ sudden enlargement}}}{\frac{4f_3 L_3 V_3^2}{2g D_3}}$$

If minor losses are neglected,

$$H = \frac{4f_1 L_1 V_1^2}{2g D_1} + \frac{4f_2 L_2 V_2^2}{2g D_2} + \frac{4f_3 L_3 V_3^2}{2g D_3}$$

If coefficient of friction is same for all pipes,

$$f_1 = f_2 = f_3 = f$$

$$H = \frac{4f}{2g} \left[\frac{L_1 V_1^2}{D_1} + \frac{L_2 V_2^2}{D_2} + \frac{L_3 V_3^2}{D_3} \right]$$

Equivalent pipe:

A compound pipe consisting of several pipes of varying diameters and lengths (i.e. pipes in series) may be replaced by a pipe of uniform diameter which is known as equivalent pipe.

For equivalent pipe, the loss of head and discharge are equal to the loss of head and discharge of compound pipe. (4)

Let L_1, L_2 and L_3 - length of compound pipe

D_1, D_2 and D_3 - Diameters of compound pipe.

We know that for compound pipe the total loss of head when neglecting minor losses

$$H = \frac{4f}{2g} \left[\frac{L_1 V_1^2}{D_1} + \frac{L_2 V_2^2}{D_2} + \frac{L_3 V_3^2}{D_3} \right] \rightarrow (1)$$

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4} D_1^2} = \frac{4Q}{\pi D_1^2} \quad V_2 = \frac{4Q}{\pi D_2^2} \quad V_3 = \frac{4Q}{\pi D_3^2}$$

Substituting the values of V_1, V_2 & V_3 in (1)

$$\begin{aligned} H &= \frac{4f}{2g} \left[\frac{L_1 \left(\frac{4Q}{\pi D_1^2} \right)^2}{D_1} + \frac{L_2 \left(\frac{4Q}{\pi D_2^2} \right)^2}{D_2} + \frac{L_3 \left(\frac{4Q}{\pi D_3^2} \right)^2}{D_3} \right] \\ &= \frac{4 \times 16 f Q^2}{\pi^2 \times 2g} \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} \right] \rightarrow (2) \end{aligned}$$

Head loss in the equivalent pipe,

$$H = \frac{4fLV^2}{2gD}$$

$$\text{where } V = \frac{4Q}{\pi D^2}$$

$$H = \frac{4fL}{2gD} \left(\frac{4Q}{\pi D^2} \right)^2 = \frac{4 \times 16 f Q^2}{\pi^2 \times 2g} \left[\frac{L}{D^5} \right]$$

Sub the value H in eq (2)

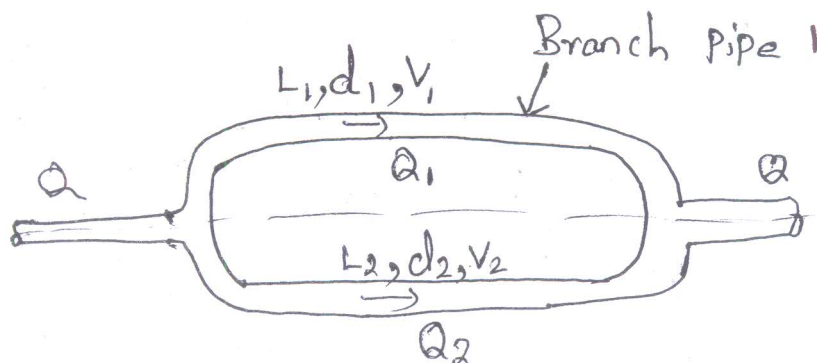
$$\frac{4 \times 16 f Q^2}{\pi^2 \times 2g} \left[\frac{L}{D^5} \right] = \frac{4 \times 16 f Q^2}{\pi^2 \times 2g} \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} \right]$$

$$\frac{L}{D^5} = \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} \right]$$

where $L = L_1 + L_2 + L_3$

The above eq is known as Darcy's eqn.

Pipes in Parallel: [To increase the discharge]



Rate of flow in the main pipe is equal ^⑤ to the sum of flow through branch pipes.

$$Q = Q_1 + Q_2$$

Head loss through each branch is same

$$h_f = h_{f1} = h_{f2}$$

$$h_f = \frac{4f_1 L_1 V_1^2}{2g D_1} = \frac{4f_2 L_2 V_2^2}{2g D_2}$$

$f_1 = f_2$ then $\frac{L_1 V_1^2}{D_1} = \frac{L_2 V_2^2}{D_2}$

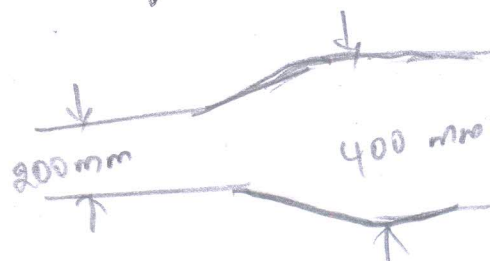
1) Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 litres/s.

Given:

$$D_1 = 200\text{mm} = 0.2\text{m}$$

$$D_2 = 400\text{mm} = 0.4\text{m}$$

$$Q = 250\text{ lit/s} = 0.25\text{ m}^3/\text{s}$$



Sol

$$A_1 = \frac{\pi}{4} \times (D_1^2) = \frac{\pi}{4} \times (0.2^2) = 0.03141 \text{ m}^2$$

$$A_2 = \frac{\pi}{4} \times (D_2^2) = \frac{\pi}{4} \times (0.4^2) = 0.12564 \text{ m}^2$$

$$Q = A_1 V_1 = A_2 V_2$$

$$\therefore V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99 \text{ m/s}$$

$$h_c = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = 1.816 \text{ m of water.}$$

2. Determine the rate of flow of water through a pipe of diameter 20cm and length 50m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4m above the centre of the pipe. The pipe considers all minor losses and take $f = 0.009$ in the formula
- $$h_f = \frac{4fLV^2}{2gd}$$

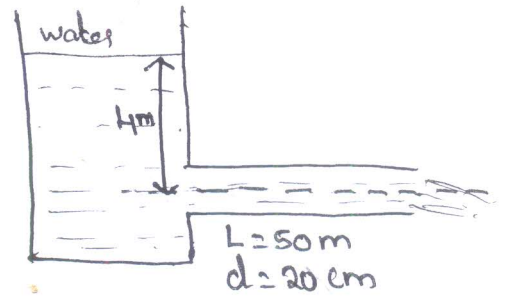
(6)

Dia of pipe, $d = 20 \text{ cm} = 0.2 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Height of water, $H = 4 \text{ m}$

Co-efficient of friction $f = 0.009$



Velocity of water in pipe (V) = ?

Sol:

By applying Bernoulli's eqn

① - Top of water surface

② - Outlet of pipe.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{all losses}$$

Consider datum line passing through the centre of pipe

$$0 + 0 + 4 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

$$4 = \frac{V_2^2}{2g} + (h_i + h_f)$$

$$(V_2 = V)$$

$$4 = \frac{V^2}{2g} + h_i + h_f + h_o$$

$$h_i = 0.5 \frac{V^2}{2g}$$

$$h_f = \frac{4fLV^2}{d \times 2g}$$

↓
head loss @ the entrance of the pipe (h_i)

h_f - Head loss due to friction in the pipe

h_o - Head loss @ the exit from a pipe

$$h_o = \frac{V^2}{2g}$$

$$4.0 = \frac{V^2}{2g} + \frac{0.5V^2}{2g} + \frac{4f \cdot L \cdot V^2}{2gd}$$

$$= \frac{V^2}{2g} \left[1 + 0.5 + \frac{4fL}{d} \right] = \frac{V^2}{2g} \left[1 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2} \right]$$

$$= \frac{V^2}{2g} [1 + 0.5 + 9] = 10.5 \times \frac{V^2}{2g}$$

$$\frac{4 \times 2g}{10.5} = V^2 \Rightarrow V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}}$$

$$V = 2.734 \text{ m/s}$$

$$V = 2.612 \text{ m/s}$$

Rate of flow

$$Q = A \times V = \frac{\pi}{4} (d^2) \times V$$

$$= \frac{\pi}{4} \times (0.2^2) \times 2.734$$

$$Q = 0.08589 \text{ m}^3/\text{s} \quad 0.04084 \text{ m}^3/\text{s}$$

C.W 11.15, 11.17

3. Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 300 mm and length 400 m. The rate of flow of water through the pipe is 300 lit/s. Consider all losses and take the value of $f = 0.008$.

Given:

$$d = 300 \text{ mm} = 0.3 \text{ m}$$

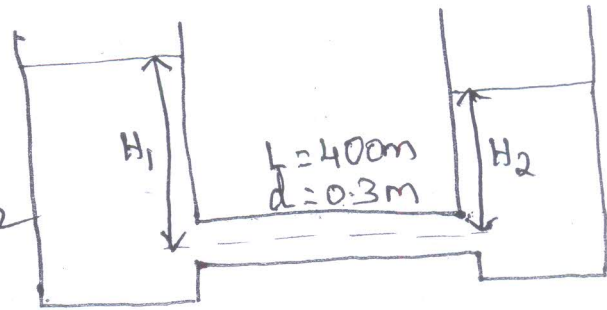
$$L = 400 \text{ m}$$

$$Q = 300 \text{ lit/s} = 0.3 \text{ m}^3/\text{s}$$

$$f = 0.008$$

Sol:

$$A = \frac{\pi}{4} (d^2) = \frac{\pi}{4} (0.3^2) \\ = 0.0706 \text{ m}^2$$



$$Q = A \times V$$

$$V = \frac{Q}{A} = \frac{0.3}{0.0706} = 4.244 \text{ m/s}$$

H_1 = Height of water in 1st tank above the centre of pipe.

By applying Bernoulli's eqn $H_2 =$

$$H_1 = H_2 + \text{Losses}$$

$$= H_2 + h_i + h_{f1} + h_o$$

$H_1 - H_2$ be the difference in elevations between water surfaces.

$$\text{Loss of head at entrance } h_i = 0.5 \frac{V^2}{2g} = \frac{0.5 \times 4.244^2}{2 \times 9.81} = 0.459 \text{ m}$$

$$\text{Loss of head due to friction } h_{f1} = \frac{4fLV^2}{2gd} = \frac{4 \times 0.008 \times 400 \times 4.244^2}{0.3 \times 2 \times 9.81} = 39.16 \text{ m}$$

$$\text{Loss of head at outlet } h_o = \frac{V^2}{2g} = \frac{4.244^2}{2 \times 9.81} = 0.918 \text{ m}$$

$$\Rightarrow H_1 = H_2 + 0.459 + 39.16 + 0.918$$

$$\therefore H_1 - H_2 = 40.537 \text{ m}$$

Hydraulic Gradient and Total Energy line:

This is very useful in the study of flow of fluids through pipes.

Hydraulic gradient line: (H.G.L):

It is defined as the line which gives the sum of pressure head ($\frac{P}{\rho g}$) and datum head (z) of a flowing fluid in a pipe with respect to some reference line.

Total Energy line (T.E.L):

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a flowing fluid in a pipe with respect to some reference line.

- 4) Determine the rate of flow of water through a pipe of diameter 20cm and length 50m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4m above the centre of the pipe. Consider all minor losses and take $f=0.009$ in the formula $h_f = \frac{4fLV^2}{2gd}$. Draw the HGL and TEL.

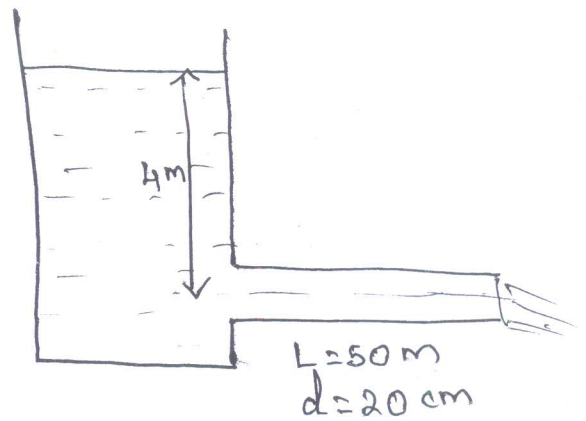
Given:-

$$d = 20 \text{ cm} = 0.2 \text{ m}$$

$$L = 50 \text{ m}$$

$$H = 4 \text{ m}$$

$$f = 0.009$$



Sol:-

By Bernoulli's eqn

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{all losses.}$$

Considering datum line passing through the centre of pipe

$$0 + 0 + H = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

$$H = \frac{V_2^2}{2g} + h_i + h_f$$

Let $V = V_2$

$$H = \frac{V^2}{2g} + h_i + h_f$$

$$H = \frac{V^2}{2g} + \frac{0.5V^2}{2g} + \frac{4fLV^2}{2gd}$$

$$= \frac{V^2}{2g} \left[1 + 0.5 + \frac{4fL}{d} \right] = \frac{V^2}{2g} \left[1 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2} \right]$$

$$= \frac{V^2}{2g} [1 + 0.5 + 9] = 10.5 \frac{V^2}{2g}$$

$$V = \sqrt{\frac{H \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/s}$$

$$Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3/\text{s}$$

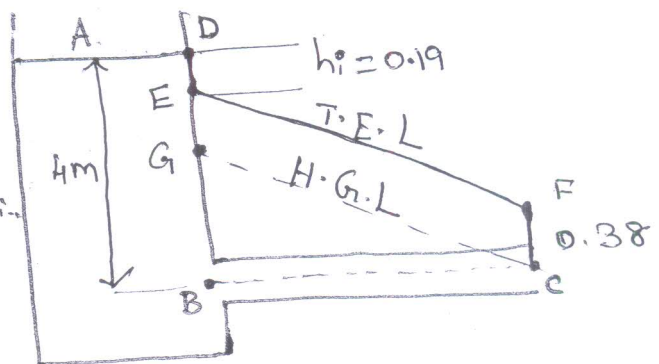
$$h_i = 0.5 \frac{V^2}{2g} = 0.5 \times \frac{2.734^2}{2 \times 9.81} = 0.19 \text{ m}$$

$$h_f = \frac{4fLV^2}{2gd} = \frac{4 \times 0.009 \times 50 \times 2.734^2}{2 \times 9.81 \times 0.2} = 3.428 \text{ m}$$

a) Total Energy Line (T.E.L).

A, B and C on the free surface of water in the tank, at inlet of the pipe and outlet respectively.

Let us find total energy at these points, taking the Centre of Pipe as reference line.



$$1. \text{ Total energy at A} = \frac{P}{\rho g} + \frac{V^2}{2g} + Z = 0 + 0 + 4 = 4 \text{ m}$$

$$2. \text{ Total energy at B} = \text{Total energy at A} - h_i = 4 - 0.19 = 3.81 \text{ m}$$

$$3. \text{ Total energy at C} = \frac{P}{\rho g} + \frac{V^2}{2g} + Z = 0 + \frac{2.734^2}{2 \times 9.81} + 0 = 0.38 \text{ m}$$

At the inlet of the pipe, it will decrease by ($h_i = 0.19 \text{ m}$) from free surface and outlet of pipe total energy is 0.38 m .

i) Point D represents total energy at A.

ii) Point E, where $DE = h_i$, represents total energy at inlet of the pipe.

iii) Point F, where $CF = 0.38$ represents total energy outlet of pipe.

Join D to E and E to F. DEF represents T.E.L.

b) Hydraulic Gradient Line (H.G.L):

9

H.G.L gives the sum of $(P/\rho + z)$ with reference to the datum line.

Hence H.G.L is obtained by subtracting $\frac{v^2}{2g}$ from total energy line (TEL).

By subtracting $\frac{v^2}{2g}$ from total energy at this point, we get point C, which lies on the Centre line of Pipe.

From C, draw a line CG parallel to EF.

Then CG represents the H.G.L.

(See 8 page notes)
in
hand written

5. The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300m, 170m and 210m and of diameters 300mm, 200mm and 400mm respectively, is 12m. Determine the rate of flow of water if Co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively, considering: (i) minor losses also
ii) neglecting minor losses.

Given data:

Difference of water level,

$$H = 12 \text{ m}$$

Length of pipe 1,

$$L_1 = 300 \text{ m} \quad d_1 = 300 \text{ mm} = 0.3 \text{ m}$$

Length of pipe 2,

$$L_2 = 170 \text{ m} \quad d_2 = 200 \text{ mm} = 0.2 \text{ m}$$

Length of pipe 3,

$$L_3 = 210 \text{ m} \quad d_3 = 400 \text{ mm} = 0.4 \text{ m}$$

$$f_1 = 0.005 \quad f_2 = 0.0052 \quad f_3 = 0.0048$$

Sol:

i) Considering Minor losses:

Let V_1 , V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

from Continuity eqn,

$$A_1 V_1 = A_2 V_2 = A_3 V_3$$

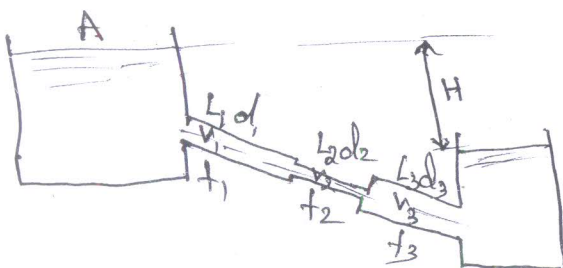
$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4} d_1^2}{\frac{\pi}{4} d_2^2} V_1 = \frac{d_1^2}{d_2^2} V_1 = \left(\frac{0.3}{0.2}\right)^2 \times V_1$$

$V_2 = 2.25 V_1$

$$V_3 = \frac{A_1 V_1}{A_3} =$$

$$= \frac{d_1^2}{d_3^2} V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1$$

$V_3 = 0.5625 V_1$



h_i (Entrance) h_c (Contraction) h_e (Exit) (10)

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

(h_a)

$$12 = \frac{0.5V_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times (2.25 V_1^2)^2}{2g}$$

$$+ \frac{4 \times 0.0052 \times 1.70 \times (2.25V_1)^2}{0.2 \times 2g} + \frac{(2.25V_1 - 0.562V_1)^2}{2g} + \frac{4 \times 0.0048 \times 210 \times (0.562V_1)^2}{0.4 \times 2g}$$

$$+ \frac{(0.5625 V_1)^2}{2g}$$

$$12 = \frac{V_1^2}{2g} \left[0.5 + 20 + 2.53 + 89.525 + 2.847 + 3.189 + 0.316 \right]$$

$$12 = \frac{V_1^2}{2g} [118.887]$$

$$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}} = 1.407 \text{ m/s}$$

Rate of flow (Q) = Area $\times$ velocity = $A_1 \times V_1$

$$= \frac{\pi}{4} \times d_1^2 \times V_1 = \frac{\pi}{4} \times (0.3)^2 \times 1.407$$

$$= 0.09945 \text{ m}^3/\text{s}$$

$$= 99.45 \text{ lit/s}$$

ii) Neglecting Minor losses:

$$H = \frac{4f_1 L_1 V_1^2}{2g d_1} + \frac{4f_2 L_2 V_2^2}{2g d_2} + \frac{4f_3 L_3 V_3^2}{2g d_3}$$

$$12 = \frac{V_1^2}{2g} \left[\frac{4f_1 L_1}{d_1} + \frac{4f_2 L_2 (2.25)^2}{d_2} + \frac{4f_3 L_3 (0.5625)^2}{d_3} \right]$$

$$12 = \frac{V_1^2}{2g} \left[\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 170 \times (2.25)^2}{0.2} \right]$$

$$+ \frac{4 \times 0.0048 \times 210 \times (0.5625)^2}{0.4} \right]$$

$$12 = \frac{V_1^2}{2g} [20 + 89.505 + 3.189]$$

$$12 = \frac{V_1^2}{2g} [112.694]$$

$$V_1 = \sqrt{\frac{2 \times 9.81 \times 12}{112.694}} = 1.445 \text{ m/s.}$$

$$\text{Discharge } Q = V_1 \times A_1 = 1.445 \times \frac{\pi}{4} (0.3)^2 = 0.1021 \text{ m}^3/\text{s} \\ = 102.1 \text{ lit/s.}$$

6. A main pipe divides into two parallel pipes which again forms one pipe as shown in fig ~~117~~. The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is $3.0 \text{ m}^3/\text{s}$. The co-efficient of friction for each parallel pipe is same and equal to 0.005.

Given:

Length of pipe 1,

$$L_1 = 2000 \text{ m}$$

Dia of pipe 1,

$$d_1 = 1.0 \text{ m}$$

Length of pipe 2,

$$L_2 = 2000 \text{ m}$$

Dia of pipe 2

$$d_2 = 0.8 \text{ m}$$

Total flow

$$Q = 3.0 \text{ m}^3/\text{s}$$

$$f_1 = f_2 = f = 0.005$$

$$Q = Q_1 + Q_2 = 3.0 \rightarrow \textcircled{1}$$

Sol:

$$\frac{4f_1 L_1 V_1^2}{2g d_1} = \frac{4f_2 L_2 V_2^2}{2g d_2}$$

$$\frac{4 \times 0.005 \times 2000 \times V_1^2}{1.0 \times 2 \times 9.81} = \frac{4 \times 0.005 \times 2000 \times V_2^2}{0.8 \times 2 \times 9.81}$$

$V_1^2 = \frac{V_2^2}{0.8}$

$$\frac{V_1^2}{1} = \frac{V_2^2}{0.8} \Rightarrow V_1^2 = \frac{V_2^2}{0.8}$$

$Q = AV$

$$V_1 = \frac{V_2}{\sqrt{0.8}} \Rightarrow \boxed{V_1 = \frac{V_2}{0.894}} \rightarrow (2)$$

$$Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} (1^2) \times \frac{V_2}{0.894}$$

$$Q_2 = \frac{\pi}{4} d_2^2 \times V_2 = \frac{\pi}{4} (0.8)^2 \times V_2 = \frac{\pi}{4} (0.64) \times V_2$$

Sub Q_1 & Q_2 in eq (1)

$$\frac{\pi}{4} \times \frac{V_2}{0.894} + \frac{\pi}{4} \times (0.64) V_2 = 3.0$$

$$0.8785 V_2 + 0.5026 V_2 = 3$$

$$V_2 (0.8785 + 0.5026) = 3$$

$$V_2 = \frac{3}{1.3811} = 2.17 \text{ m/s}$$

Sub V_2 in eq (2)

(12)

$$V_1 = \frac{V_2}{0.894} = \frac{2.17}{0.894} = 2.427 \text{ m/s}$$

$$Q_1 = A_1 V_1 = \frac{\pi}{4} \times d_1^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427 = 1.906 \text{ m}^3/\text{s}$$

$$Q = Q - Q_1 = 3.0 - 1.906 = 1.094 \text{ m}^3/\text{s}$$

7) A Pipe of diameter 20 cm and length 2000 m connected two reservoirs, having difference of water levels as 20 m. Determine the discharge through the pipe. [If an additional pipe of diameter 20 cm and length 1200 m is attached to the last 1200 m length of the existing pipe, find the increase in the discharge. Take $f = 0.015$ and neglect minor loss]

Given:

Dia of Pipe, $d = 20 \text{ cm} = 0.2 \text{ m}$

Length of Pipe, $L = 2000 \text{ m}$

Difference of water levels $H = 20 \text{ m}$

co-efficient of friction $f = 0.015$

1st Case: when a single pipe connects the two reservoirs

$$H = \frac{4fLV^2}{2gd}$$

$$Q = AV \Rightarrow V = \frac{Q}{A}$$

$$H = \frac{4fL}{2gd} \left[\frac{Q}{\frac{\pi}{4} d^2} \right]^2$$

$$\therefore V = \frac{Q}{\frac{\pi}{4} d^2}$$

$$= \frac{32 f L Q^2}{\pi^2 g d^5}$$

$$20 = \frac{32 \times 0.15 \times 2000 \times Q^2}{\pi^2 \times 9.81 \times (0.2)^5} = 30985.07 Q^2$$

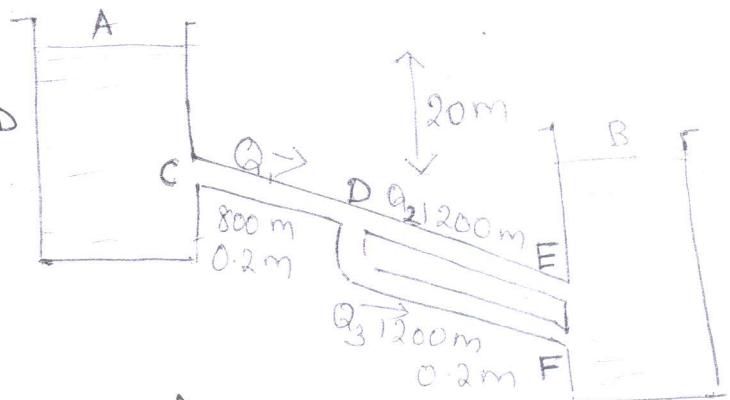
$$\therefore Q = \sqrt{\frac{20}{30985.07}} = 0.0254 \text{ m}^3/\text{s}$$

2nd Case:

Let Q_1 = Discharge through pipe CD

Q_2 = Discharge through pipe DE

Q_3 = Discharge through pipe DF



Length of pipe CD, $L_1 = 800\text{m}$ $d_1 = 0.2\text{m}$

Length of pipe DE, $L_2 = 1200\text{m}$ $d_2 = 0.2\text{m}$

Length of pipe DF, $L_3 = 1200\text{m}$ $d_3 = 0.2\text{m}$

Since length and diameter of pipes DE & DF are equal
 Q_2 will be equal to Q_3

Like parallel pipe

$$Q_1 = Q_2 + Q_3 = Q_2 + Q_2 = 2Q_2 \quad \therefore Q_2 = \frac{Q_1}{2}$$

By applying Bernoulli's eqn to points A and B and taking the flow through CDE,

$$20 = \frac{4fL_1 \cdot V_1^2}{2gd_1} + \frac{4fL_2 V_2^2}{2gd_2} \rightarrow \textcircled{1}$$

$$V_1 = \frac{Q_1}{A} = \frac{Q_1}{\frac{\pi}{4}(d_1^2)} = \frac{Q_1}{\frac{\pi}{4}(0.2^2)} = \frac{4Q_1}{\pi \times 0.04}$$

$$V_2 = \frac{Q_2}{A} = \frac{Q_2}{\frac{\pi}{4} \times (d_2^2)} = \frac{Q_2}{\frac{\pi}{4}(0.2^2)} = \frac{4 \times \frac{Q_1}{2}}{\pi \times 0.04} = \frac{2Q_1}{\pi \times 0.04}$$

Sub V_1 & V_2 in eq^①

$$20 = \frac{4 \times 0.015 \times 800}{0.2 \times 2 \times 9.81} \times \left(\frac{4Q_1}{\pi \times 0.04} \right)^2 + \frac{4 \times 0.015 \times 1200}{0.2 \times 2 \times 9.81} \left(\frac{2Q_1}{\pi \times 0.04} \right)^2$$

$$20 = 12394 Q_1^2 + 4647 Q_1^2 = 17041 Q_1^2$$

$$Q_1 = \sqrt{\frac{20}{17041}} = 0.0342 \text{ m}^3/\text{s}$$

Increase in discharge $= Q_1 - Q = 0.0342 - 0.0254$

$$= 0.0088 \text{ m}^3/\text{s}$$

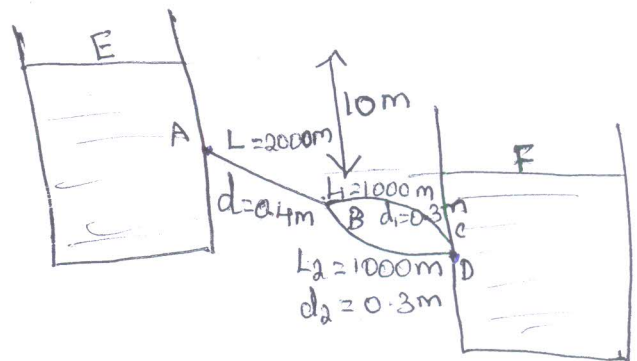
8) A Pipe of diameter 0.4m and of length 2000m is connected to a reservoir at one end. The other end of the pipe is connected to a Junction from which two pipes of lengths 1000m and diameter 300mm run in parallel. These parallel pipes are connected to another reservoir, which is having level of water 10m below the water level of the above reservoir. Determine the total discharge if $f=0.015$. Neglect minor losses.

Given:

Dia of pipe, $d = 0.4\text{m}$

Length of pipe $L = 2000\text{m}$

Dia of Parallel pipes $d_1 = d_2 = 300\text{mm} = 0.3\text{m}$



Length of Parallel pipes, $L_1 = L_2 = 1000\text{m}$

Difference of water level in two reservoirs $H = 10\text{m}$, $f = 0.015$

Sol: Applying Bernoulli's equation to points E & F.

$$10 = \frac{4fLV^2}{d \times 2g} + \frac{4f \times L_1 \times V_1^2}{2g d_1}$$

$$= \frac{4 \times 0.015 \times 2000 \times V^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.015 \times 1000 \times V_1^2}{0.3 \times 2 \times 9.81}$$

$$= 15.29 V^2 + 10.19 V_1^2 \rightarrow \text{--- (1)}$$

from Continuity eqn

(14)

Discharge through AB = Discharge through BC + Discharge through BD

$$\frac{\pi}{4} d^2 \times V = \frac{\pi}{4} d_1^2 \times V_1 + \frac{\pi}{4} d_1^2 \times V_2$$

Since $\frac{d_1 = d_2}{L_1 = L_2}$

$\therefore V_1 = V_2$

$$\frac{\pi}{4} d^2 V = \frac{\pi}{4} d_1^2 \times V_1 + \frac{\pi}{4} d_1^2 \times V_1$$

$$\frac{\pi}{4} d^2 V = 2 \times \frac{\pi}{4} d_1^2 \times V_1$$

$$d^2 V = 2 d_1^2 V_1$$

$$(0.4)^2 \times V = 2 \times (0.3)^2 V_1$$

$$0.16 V = 0.18 V_1$$

$$V_1 = \frac{0.16}{0.18} V = 0.888 V$$

Sub this value of V_1 in eq (1) we get

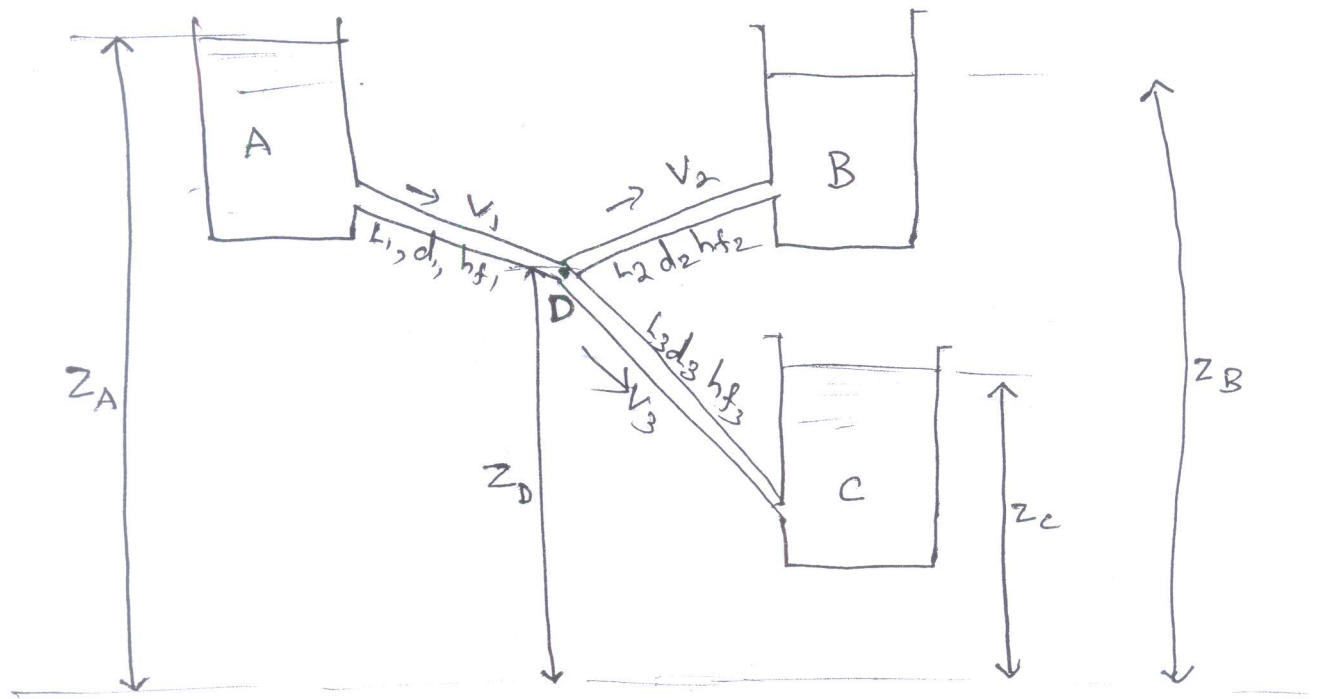
$$10 = 15.29 V^2 + 10.19 \times (0.888)^2 V^2$$

$$10 = 15.29 V^2 + 8.035 V^2 = 23.325 V^2$$

$$V = \sqrt{\frac{10}{23.325}} \quad \boxed{V = 0.654 \text{ m/s}}$$

$$Q = A \times V = \frac{\pi}{4} d^2 \times 0.654 = \frac{\pi}{4} \times (0.4^2) \times 0.654$$
$$Q = 0.0822 \text{ m}^3/\text{s}$$

⇒ Flow through Branched pipe: [Eg: 11.42]



for flow A to D from Bernoulli's eqn

$$z_A = z_D + \frac{P_D}{\rho g} + h_{f1}$$

for flow D to B from Bernoulli's eqn

$$z_D + \frac{P_D}{\rho g} = z_B + h_{f2}$$

for flow D to C from Bernoulli's eqn

$$z_D + \frac{P_D}{\rho g} = z_C + h_{f3}$$

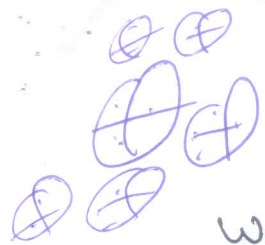
from continuity eqn

Discharge through AD = DB + DC

$$\frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2 + \frac{\pi}{4} d_3^2 V_3$$

$$d_1^2 V_1 = d_2^2 V_2 + d_3^2 V_3$$

Darcy's - Weisbach equation:



A pipe is a closed conduit through which the fluid flows under pressure.

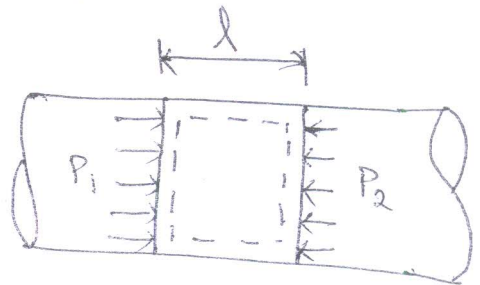
When the fluid flows through the piping system, some of the potential energy is lost due to friction.

The various viscous friction effects associated with fluid are proportional to

- * Length of pipe, L
- * The wetted perimeter, P
- * V^n

V - Avg velocity of flow

n - Index varies from 1.5 to 2



Consider a horizontal pipe of cross sectional area A carrying a fluid with a mean velocity V .

Let P_1 and P_2 be the intensities of pressure at section 1 and 2 respectively.

By applying Bernoulli's eqn between sec (1) & (2)

$$\frac{V_1^2}{2g} + \frac{P_1}{w} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{w} + Z_2 + h_f$$

Since $V_1 = V_2 = V$ and $z_1 = z_2$

$$\frac{P_1}{w} = \frac{P_2}{w} + h_f$$

loss of head $h_f = \frac{P_1}{w} - \frac{P_2}{w}$ $h_f = \frac{P_1 - P_2}{w}$

Let f' be the frictional resistance per unit area at unit velocity,

$$\begin{aligned}\text{Frictional resistance} &= f' \times \text{Area} \times V^n \\ &= f' \times PL \times V^n\end{aligned}$$

P = wetted
Perimeter of the
pipe

$$\text{Pressure force of section 1} = P_1 A$$

$$\text{Pressure force of section 2} = P_2 A$$

$$P_1 A = P_2 A + \text{frictional resistance}$$

$$(P_1 - P_2) A = f' \times PL \times V^n$$

$$(P_1 - P_2) = f' \times \frac{PL}{A} \times V^n$$

$\div$ both sides by specific weight w

$$\frac{P_1 - P_2}{w} = \frac{f'}{w} \times \frac{PL}{A} \times V^n$$

Sub h_f value

$$h_f = \frac{f'}{w} \times \frac{PL}{A} \times V^n$$

$$\frac{P_1 - P_2}{w} = h_f$$

Laminar and Turbulent flow:

(15)

Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel.

Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way.

$$Re = \frac{\rho V D}{\mu}$$

ρ - Density of fluid

V - Avg velocity of fluid

D - Diameter of pipe

μ - Viscosity of fluid.

$Re < 2000 \rightarrow$ Laminar flow

$2000 < Re < 4000 \rightarrow$ Transition flow

$Re > 4000 \rightarrow$ Turbulent flow.

Since $V_1 = V_2 = V$ and $z_1 = z_2$

$$\frac{P_1}{w} = \frac{P_2}{w} + h_f$$

loss of head $h_f = \frac{P_1}{w} - \frac{P_2}{w}$ $h_f = \frac{P_1 - P_2}{w}$

Let f' be the frictional resistance per unit area at unit velocity,

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$$(P_1 - P_2) A = f' \times PL \times V^n$$

$$(P_1 - P_2) = f' \times \frac{PL}{A} \times V^n$$

$\div$ both sides by specific weight w

$$\frac{P_1 - P_2}{w} = \frac{f'}{w} \times \frac{PL}{A} \times V^n$$

Sub h_f value

$$h_f = \frac{f'}{w} \times \frac{PL}{A} \times V^n$$

$$\frac{P_1 - P_2}{w} = h_f$$

Since $V_1 = V_2 = V$ and $z_1 = z_2$

$$\frac{P_1}{w} = \frac{P_2}{w} + h_f$$

loss of head $h_f = \frac{P_1}{w} - \frac{P_2}{w}$ $h_f = \frac{P_1 - P_2}{w}$

Let f' be the frictional resistance per unit area at unit velocity,

$$\begin{aligned}\text{Frictional resistance} &= f' \times \text{Area} \times V^n \\ &= f' \times PL \times V^n\end{aligned}$$

P = wetted
Perimeter of the
pipe

$$\text{Pressure force of section 1} = P_1 A$$

$$\text{Pressure force of section 2} = P_2 A$$

$$P_1 A = P_2 A + \text{frictional resistance}$$

$$(P_1 - P_2) A = f' \times PL \times V^n$$

$$(P_1 - P_2) = f' \times \frac{PL}{A} \times V^n$$

$\div$ both sides by specific weight w

$$\frac{P_1 - P_2}{w} = \frac{f'}{w} \times \frac{PL}{A} \times V^n$$

Sub h_f value

$$h_f = \frac{f'}{w} \times \frac{PL}{A} \times V^n$$

$$\frac{P_1 - P_2}{w} = h_f$$

For equilibrium, the summation of all these forces in the x -direction must be equal to Zero.

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x} dx \right) \pi r^2 - \tau \cdot 2\pi r \cdot dx = 0$$

Annotations: pressure force, pressure force right side, volume of fluid element, shear stress

$$p\pi r^2 - p\pi r^2 - \left(\frac{\partial p}{\partial x} dx \right) \pi r^2 - \tau \cdot 2\pi r \cdot dx = 0$$

$$\left(-\frac{\partial p}{\partial x} dx \right) \pi r^2 - \tau \cdot 2\pi r \cdot dx = 0$$

Dividing the above equation throughout by volume of the fluid element $\pi r^2 dx$

$$\frac{\left(-\frac{\partial p}{\partial x} dx \right) \pi r^2}{\pi r^2 dx} - \frac{\tau \cdot 2\pi r \cdot dx}{\pi r^2 dx} = 0$$

$\frac{\partial p}{\partial x} = \text{pressure gradient}$

$$-\frac{\partial p}{\partial x} - \frac{2\tau}{r} = 0$$

$$-\frac{2\tau}{r} = \frac{\partial p}{\partial x} \quad (-)$$

$$\begin{pmatrix} (-) \times (-) \\ -x = y \end{pmatrix} \Rightarrow x = -y$$

$$\boxed{\tau = -\frac{\partial p}{\partial x} \times \frac{r}{2}} \quad \text{--- (1)}$$

eqn (1) shows that the shear stress τ varies linearly along the radius of pipe. It is evident from eqn that the shear stress will be zero at the centre of the pipe as $r=0$ & maximum of pipe wall as $r=R$. Shown in fig shows the shear stress distribution across the pipe for laminar flow.

From newton's law of viscosity for laminar flow,

$$\tau = \mu \frac{du}{dy} = - \mu \frac{du}{dr} \quad \text{--- (2)}$$

$R=y$
 $r=R$

(-ve) sign shows that u decreases as y increases)

eqn ① & ②

$$\mu \frac{du}{dr} = - \frac{\partial P}{\partial x} \frac{r}{2}$$

$$\frac{du}{dr} = - \frac{1}{\mu} \frac{\partial P}{\partial x} \frac{r}{2}$$

integrating the above eqn with respect to r

$$\int \frac{du}{dr} = - \frac{1}{\mu} \frac{\partial P}{\partial x} \int \frac{r}{2}$$

$$u = - \frac{1}{\mu} \frac{\partial P}{\partial x} \frac{r^2}{2 \times 2} + C$$

$$u = - \frac{1}{4\mu} \frac{\partial P}{\partial x} r^2 + C \quad \text{--- (3)}$$

where, $C = \text{constant of integration}$

The value of ' C ' can be determined from the following boundary condition.

$$u = 0 \text{ @ } r = R$$

Substituting this Boundary Condition in eqn (3)

$$0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + C$$

$$C = -\frac{1}{4\mu} \left(\frac{\partial p}{\partial x} \right) R^2$$

Hence, the velocity distribution is given by

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 + C$$

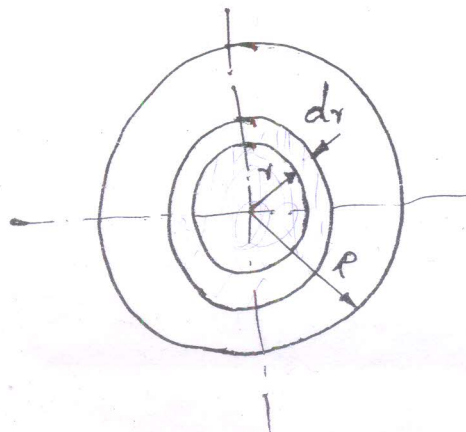
$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \left(\frac{\partial p}{\partial x} \right) R^2$$

$$u = -\frac{1}{4\mu} \left(\frac{\partial p}{\partial x} \right) [R^2 - r^2] \quad \text{--- (4)}$$

Maximum velocity is obtained when $r=0$

$$u_{\max} = -\frac{1}{4\mu} \left(\frac{\partial p}{\partial x} \right) [R^2 - 0]$$

$$u_{\max} = -\frac{1}{4\mu} \left(\frac{\partial p}{\partial x} \right) R^2$$



The quantity of fluid flowing per second through the pipe, dQ is obtained by considering the flow through a circular ring element of radius r and thickness dr as shown in fig.

$$dQ = \overset{\text{Area}}{2\pi r} \overset{\text{thickness}}{dr} \times \overset{\text{velocity}}{u}$$

circumference = $2\pi r$

thickness = dr

velocity = u

$$= 2\pi r dr \times \left[-\frac{1}{4\mu} \left(\frac{\partial p}{\partial x} \right) (R^2 - r^2) \right]$$

integrating the above eqn.

$$Q = - \frac{2\pi r}{4\mu} dr \times \left(\frac{\partial p}{\partial x} \right) (R^2 - r^2)$$

$$= - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \int_0^R (R^2 - r^2) r dr$$

$$= - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \int_0^R (R^2 r - r^3) dr$$

$$= - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \left[R^2 \frac{r^2}{2} - \frac{r^4}{4} \right]_0^R$$

$$= - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \left[R^2 \times \frac{R^2}{2} - \frac{R^4}{4} \right]$$

$$= - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \frac{R^4}{2} - \frac{R^4}{4}$$

$$= - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \frac{4R^4 - 2R^4}{8}$$

$$= - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \frac{2R^4}{8} = - \frac{\pi}{2\mu} \left(\frac{\partial p}{\partial x} \right) \frac{R^4}{4}$$

$$= \frac{-\pi}{2\mu} \left(\frac{\partial P}{\partial x} \right) \frac{R^4}{4}$$

$$R = \frac{D}{2} \Rightarrow R^4 = \left(\frac{D}{2} \right)^4$$

$$= \frac{-\pi}{8\mu} \left(\frac{\partial P}{\partial x} \right) R^4 \Rightarrow \left(\frac{D}{2} \right)^4$$

$$R^4 = \frac{D^4}{16}$$

$$= \frac{-\pi}{8\mu} \left(\frac{\partial P}{\partial x} \right) \times \frac{D^4}{16}$$

$$16 \times 8 = 128$$

$$Q = \frac{-\pi}{128\mu} \left(\frac{\partial P}{\partial x} \right) \times D^4 \quad \text{--- (5)}$$

Rearranging the above eqn.

$$\left(-\frac{\partial P}{\partial x} \right) = \frac{128\mu Q}{\pi D^4}$$

The term $\left(-\frac{\partial P}{\partial x} \right)$ represents pressure drop per unit length of the pipe and may be written as $\frac{P_1 - P_2}{L} = \frac{-\partial P}{\partial x}$

$$\frac{P_1 - P_2}{L} = \frac{128\mu Q}{\pi D^4} \quad \text{--- (6)}$$

The mean velocity of flow V , which is equal to (Q/A) (or $(Q/\pi R^2)$) is obtained from eqn (5)

$$Q = A \times V$$

$$V = u$$

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} D^2} = \frac{\frac{-\pi}{128\mu} \left(\frac{\partial P}{\partial x} \right) D^4}{\frac{\pi}{4} D^2}$$

$$V = -\frac{1}{32\mu} \left(\frac{\partial P}{\partial x} \right) D^2$$

$$\frac{-\partial P}{\partial x} = \frac{32\mu V}{D^2}$$

$$\frac{P_1 - P_2}{L} = \frac{32 \mu V}{D^2} \quad \text{--- (7)}$$

From eqns (6), (7)

$$P_1 - P_2 = \frac{32 \mu V L}{D^2} = \frac{128 \mu Q L}{\pi D^4}$$

This equation is called as Hagen - poiseuille equation for laminar flow in the circular pipes.

The Hagen - poiseuille's equation is sometimes also written in terms of loss of head, h_L as follows:

$$h_L = \frac{P_1 - P_2}{w} = \frac{32 \mu V L}{w D^2}$$

comparing this equation with that of Darcy - weisbach eqn.

$$h_L = \frac{f L V^2}{2gD} = \frac{32 \mu V L}{w D^2}$$

$$f = \frac{32 \times 2g \mu V L}{w D^2 \times L V^2}$$

$$= \frac{64 g \mu}{w D V}$$

$$f = \frac{64 g \mu}{e g D V} = \frac{64 \mu}{e V D} \quad \left\{ \because Re = \frac{e V D}{\mu} \right.$$

$$f = \frac{64}{Re}$$

$Re \leftarrow$ Reynolds number

This eqn relationship b/w friction factor f & Reynolds number Re for laminar flow through a circular pipe.

$Re = \frac{e V D}{\mu}$

$\frac{V \times D}{\mu}$

$$e = \frac{w}{g}$$

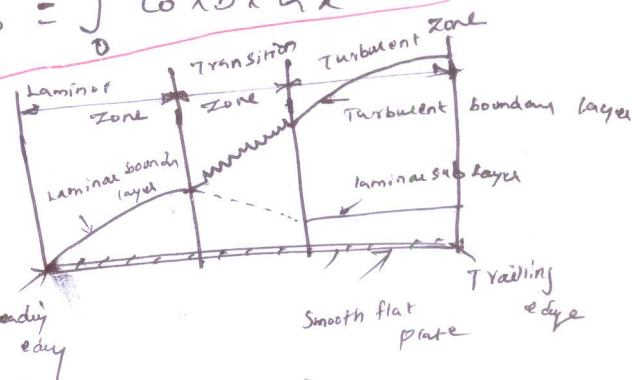
$$w = e g$$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial}{\partial x} \left[\int_0^{\delta} \frac{u}{U} \left[1 - \frac{u}{U} \right] dy \right]$$

local co-efficient of Drag [C_D^*]

$$C_D^* = \frac{\tau_0}{\frac{1}{2} \rho U^2}$$

$$F_D = \int_0^L \tau_0 \times b \times dx$$



Average co-efficient of Drag [C_D]:

$$C_D = \frac{F_D}{\frac{1}{2} \rho A U^2}$$

$$F_D = \int_0^L \tau_0 \times b \times dx$$

A = Area of the Surface

U = Free stream velocity

ρ = Mass density of fluid

① For the velocity profile for laminar boundary layer flows given as

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$\delta \neq \text{delta}$

Find an expression for boundary layer thickness (δ), Shear stress (τ_0) & co-efficient of drag [C_D] in terms of Reynold numbers.

Given:

$$\text{The velocity distribution } \frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

To Find

① Boundary layer thickness (δ)

② Shear stress (τ_0)

③ co-efficient of drag [C_D]

④

Solution

$$\begin{aligned}\frac{\tau_0}{\rho v^2} &= \frac{d}{dx} \left[\int_0^{\delta} \frac{u}{v} \left(1 - \left(\frac{u}{v} \right) \right) dy \right] \quad \left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2} \right] \\&= \frac{2}{\delta^2} \left[\int_0^{\delta} \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right] \left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right) \right] dy \right] \Rightarrow dy \\&= \frac{2}{\delta^2} \left[\int_0^{\delta} \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4} \right] dy \right] \\&= \frac{2}{\delta^2} \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{3y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4} \right] dy \\&= \frac{2}{\delta^2} \left[\frac{2y^2}{2\delta} - \frac{3y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4} \right]_0^{\delta} \\&= \frac{2}{\delta^2} \left[\frac{2\delta^2}{2\delta} - \frac{3\delta^3}{3\delta^2} + \frac{4\delta^4}{4\delta^3} - \frac{\delta^5}{5\delta^4} \right] \\&= \frac{2}{\delta^2} \left[\delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5} \right] \\&= \frac{2}{\delta^2} \left[\frac{3\delta - 5\delta}{3} + \frac{5\delta - \delta}{5} \right] \\&= \frac{2}{\delta^2} \left[\frac{-2\delta}{3} + \frac{4\delta}{5} \right] \\&= \frac{2}{\delta^2} \left[\frac{-10\delta + 12\delta}{15} \right] \\&= \frac{2}{\delta^2} \left[\frac{2\delta}{15} \right]\end{aligned}$$

$$\frac{\tau_0}{\rho v^2} = \frac{2}{15} \frac{2}{\delta^2} [\delta]$$

$$\frac{\tau_0}{\rho v^2} = \frac{2}{15} \frac{2}{dx} (\delta)$$

$$\tau_0 = \rho U^2 \times \frac{2}{15} \frac{\partial}{\partial x} [f]$$

$$\tau_0 = \frac{2}{15} \rho U^2 \frac{d}{dx} (f)$$

(3)

$$\tau_0 = \frac{2}{15} \rho U^2 \frac{\partial [f]}{\partial x}$$

— (a)

The shear stress at the boundary in laminar flow is also given by newton's law of viscosity as

$$\tau_0 = \mu \left(\frac{du}{dy} \right)_{y=0}$$

— (2)

$$\frac{u}{U} = \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right]$$

$$u = U \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} \right]$$

$$\frac{du}{dy} = U \left[\frac{2}{\delta} - \frac{2y}{\delta^2} \right]$$

$$\left(\frac{du}{dy} \right)_{y=0} = U \left[\frac{2}{\delta} - \frac{2(0)}{\delta^2} \right]$$

$$\left(\frac{du}{dy} \right)_{y=0} = \frac{2U}{\delta}$$

Sub values in eqn (2)

$$\tau_0 = \mu \times \frac{2U}{\delta} = \frac{2\mu U}{\delta}$$

$$\tau_0 = \frac{2\mu U}{\delta}$$

— (3)

Equating this value of τ_0 given by (a) & (3)

$$= 5.48 \sqrt{\frac{\mu x \times x}{\rho U \times x}}$$

$$= 5.48 \sqrt{\frac{\mu x^2}{\rho U \times x}}$$

$$Re_x = \frac{\rho U x}{\mu} \quad \checkmark$$

$$= 5.48 \sqrt{\frac{x^2}{Re_x}} = 5.48 \frac{x}{\sqrt{Re_x}}$$

$$\boxed{f = 5.48 \frac{x}{\sqrt{Re_x}}}$$

② shear stress (τ_0) in terms of Reynold number!

from Qn ③

$$\tau_0 = \frac{2\mu U}{f} = \frac{2\mu U}{5.48 \frac{x}{\sqrt{Re_x}}}$$

$$= \frac{2\mu U \sqrt{Re_x}}{5.48 x} = 0.365 \frac{\mu U}{x} \sqrt{Re_x} = \tau_0$$

③ co-efficient of Drag [C_D] drag force

$$C_D = \frac{F_D}{\frac{1}{2} \rho A U^2}$$

$$F_D = \int_0^L \tau_0 \times b \times dx = \int_0^L 0.365 \frac{\mu U}{x} \sqrt{Re_x} \times b \times dx$$

$$= 0.365 \int_0^L \frac{\mu U}{x} \sqrt{\frac{\rho U x}{\mu}} \times b \times dx$$

$$Re = \frac{\rho U x}{\mu} \quad \checkmark \quad \sqrt{x} \times \sqrt{x} = x$$

$$= 0.365 \int_0^L \frac{\mu U}{\sqrt{x} \times \sqrt{x}} \sqrt{\frac{\rho U x}{\mu}} \times b \times dx$$

(6)

$$= 0.365 \int_0^L \frac{\mu V}{\sqrt{x}} \sqrt{\frac{eV}{\mu}} x b x dx$$

$$= 0.365 \int_0^L \mu V \sqrt{\frac{eV}{\mu}} x \frac{1}{\sqrt{x}} x b x dx$$

$$= 0.365 \mu V \sqrt{\frac{eV}{\mu}} x b x \int_0^L \frac{1}{\sqrt{x}} dx$$

$$= 0.365 \mu V \sqrt{\frac{eV}{\mu}} x b x \int_0^L x^{-1/2} dx \quad \therefore \int \frac{x^{-1/2+1}}{-1/2+1} = \frac{x^{1/2}}{1/2}$$

$$= 0.365 \mu V \sqrt{\frac{eV}{\mu}} x b x \left[\frac{x^{1/2}}{1/2} \right]_0^L$$

$$x^{1/2} = (L)^{1/2} = \sqrt{L}$$

$$= 0.325 \times 2 \mu V \sqrt{\frac{eV}{\mu}} x b x \sqrt{L} \quad \leftarrow$$

$$\int \frac{x^{n+1}}{n+1} = \int \frac{x^{-1/2+1}}{-1/2+1}$$

$$F_D = 0.736 \mu V \sqrt{\frac{eVL}{\mu}}$$

$$C_D = \frac{0.736 \mu V \sqrt{\frac{eVL}{\mu}}}{\frac{1}{2} \epsilon A V^2}$$

$$= \frac{0.736 \mu V}{\frac{1}{2} \epsilon A V^2} \sqrt{\frac{eVL}{\mu}}$$

Area of plate = $L \times b$

$$= \frac{2 \times 0.736 \mu}{\epsilon A V} \sqrt{\frac{eVL}{\mu}}$$

$$= \frac{1.466 \mu}{\epsilon \times L \times b \times V} \sqrt{\frac{eVL}{\mu}}$$

$$= \frac{1.46 \mu}{\epsilon L V} \sqrt{\frac{eVL}{\mu}}$$

$$= 5.48 \sqrt{\frac{\mu x \times x}{\rho U \times x}}$$

$$= 5.48 \sqrt{\frac{\mu x^2}{\rho U \times x}}$$

$$Re_x = \frac{\rho U x}{\mu} \quad \checkmark$$

$$= 5.48 \sqrt{\frac{x^2}{Re_x}} = 5.48 \frac{x}{\sqrt{Re_x}}$$

$$\boxed{f = 5.48 \frac{x}{\sqrt{Re_x}}}$$

② shear stress (τ_0) in terms of Reynold number!

from Qn ③

$$\tau_0 = \frac{2\mu U}{f} = \frac{2\mu U}{5.48 \frac{x}{\sqrt{Re_x}}}$$

$$= \frac{2\mu U \sqrt{Re_x}}{5.48 x} = 0.365 \frac{\mu U}{x} \sqrt{Re_x} = \tau_0$$

③ co-efficient of Drag [C_D]

$$C_D = \frac{F_D}{\frac{1}{2} \rho A U^2}$$

$$F_D = \int_0^L \tau_0 \times b \times dx = \int_0^L 0.365 \frac{\mu U}{x} \sqrt{Re_x} \times b \times dx$$

$$= 0.365 \int_0^L \frac{\mu U}{x} \sqrt{\frac{\rho U x}{\mu}} \times b \times dx$$

$$Re = \frac{\rho U x}{\mu} \quad \sqrt{x} \times \sqrt{x} = x$$

$$= 0.365 \int_0^L \frac{\mu U}{\sqrt{x} \times \sqrt{x}} \sqrt{\frac{\rho U x}{\mu}} \times b \times dx$$

$$= 5.48 \sqrt{\frac{\mu x \times x}{\rho U \times x}}$$

$$= 5.48 \sqrt{\frac{\mu x^2}{\rho U \times x}}$$

$$Re_x = \frac{\rho U x}{\mu} \quad \checkmark$$

$$= 5.48 \sqrt{\frac{x^2}{Re_x}} = 5.48 \frac{x}{\sqrt{Re_x}}$$

$$\boxed{f = 5.48 \frac{x}{\sqrt{Re_x}}}$$

② shear stress (τ_0) in terms of Reynold number!

from Qn ③

$$\tau_0 = \frac{2\mu U}{f} = \frac{2\mu U}{5.48 \frac{x}{\sqrt{Re_x}}}$$

$$= \frac{2\mu U \sqrt{Re_x}}{5.48 x} = 0.365 \frac{\mu U}{x} \sqrt{Re_x} = \tau_0$$

③ co-efficient of Drag [C_D] drag force

$$C_D = \frac{F_D}{\frac{1}{2} \rho A U^2}$$

$$F_D = \int_0^L \tau_0 \times b \times dx = \int_0^L 0.365 \frac{\mu U}{x} \sqrt{Re_x} \times b \times dx$$

$$= 0.365 \int_0^L \frac{\mu U}{x} \sqrt{\frac{\rho U x}{\mu}} \times b \times dx$$

$$Re = \frac{\rho U x}{\mu} \quad \checkmark \quad \sqrt{x} \times \sqrt{x} = x$$

$$= 0.365 \int_0^L \frac{\mu U}{\sqrt{x} \times \sqrt{x}} \sqrt{\frac{\rho U x}{\mu}} \times b \times dx$$

Dimensions and its units:

S.NO	Physical quantity	Symbol	Dimensions
1.	Fundamental quantities a) Mass b) Length c) Time	M L T	M L T
2.	Geometric quantities (a) Area m^2 b) Volume m^3 c) Moment of inertia m^4	A m^2 V m^3 I m^4	L^2 L^3 L^4
3.	Kinetic quantities a) Velocity m/s b) Angular velocity rad/s c) Acceleration m/s^2 d) Angular acceleration rad/s^2 e) Gravity m/s^2 f) Discharge m^3/s g) Kinetic viscosity m^2/s	V $m s^{-1}$ ω a $m s^{-2}$ α g $m s^{-2}$ Q $m^3 s^{-1}$ ν $m^2 s^{-1}$	LT^{-1} T^{-1} LT^{-2} T^{-2} LT^{-2} $L^3 T^{-1}$ $L^2 T^{-1}$
4.	Dynamic quantities a) Force $M \times m/s^2$ b) Weight $M \times \frac{Length}{Time^2}$ Speed $N = m \times g =$	F $M L T^{-2}$ W $M L T^{-2}$ N	MLT^{-2} MLT^{-2} T

Bulk modulus (K) = $ML^{-1}T^{-2}$

rad = 1

c) Specific weight

d) Density

e) Dynamic viscosity

f) Work

g) Energy

h) Power

i) Torque

j) Momentum

$$\mu \quad \frac{\text{kg}}{\text{m}^2 \times \text{s}} \quad \text{ML}^{-2} \text{T}^{-1}$$

$$\rho \quad \frac{\text{kg}}{\text{m}^3} \quad \text{ML}^{-3}$$

$$\mu \quad \frac{\text{kg}}{\text{m} \cdot \text{s}} \quad \text{ML}^{-1} \text{T}^{-1}$$

$$W \quad \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \quad \text{ML}^2 \text{T}^{-2}$$

$$E \quad \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \quad \text{ML}^2 \text{T}^{-2}$$

$$P \quad \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} \quad \text{ML}^2 \text{T}^{-3}$$

$$T \quad \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \quad \text{ML}^2 \text{T}^{-2}$$

$$M \quad \frac{\text{kg} \cdot \text{m}}{\text{s}} \quad \text{MLT}^{-1}$$

$$\text{shear stress} = \frac{\text{N}}{\text{m}^2} = \frac{\text{N} \times \text{g}}{\text{m}^2} = \frac{\text{kg} \times \frac{\text{m}}{\text{s}^2}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2} \Rightarrow \text{ML}^{-1} \text{T}^{-2}$$

Dimensional

$$\text{Homogeneity: } \frac{\text{kg} \times \frac{\text{m}}{\text{s}^2}}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = \text{ML}^{-1} \text{T}^{-2}$$

The law of Fourier principle of dimension homogeneity states "an equation which expresses a physical phenomenon of fluid flow should be algebraically correct and dimensionally homogeneous".

(Dimensionally homogeneous means, the dimension of the term of left hand side should be same as the dimension of the term on right hand side.)

Let us consider an equation

$$V = \sqrt{2gH}$$

L.H.S

$$V = \frac{\text{m}}{\text{s}} = \frac{\text{L}}{\text{T}} = \text{LT}^{-1}$$

$$\sqrt{2gH} = \sqrt{\frac{\text{L}}{\text{T}^2} \times \text{L}} = \sqrt{\frac{\text{L}^2}{\text{T}^2}} = \frac{\text{L}}{\text{T}} = \text{LT}^{-1}$$

Uses of Dimensional homogeneity :

- i) To check the dimensional homogeneity of the given equation.
- ii) To determine the dimension of a physical variable.
- iii) To ~~convert~~^{convert} units from one system to another through dimensional homogeneity.
- iv) It is a step towards dimensional analysis.

Methods of Dimensional Analysis :

- i) Rayleigh's method
- ii) Buckingham Π - Theorem.

i) Rayleigh's method :

In this method, the expression is determined for a variable depending upon maximum three or four variables only.

If the number of independent variables becomes more than four, it is very difficult to find the expression for the dependent variable. So, a functional relationship between variables is expressed in exponential form or equation.

Steps involved in Rayleigh's method:

1. First, the functional relationship is written with the given data.

Consider X as a variable which depends on $X_1, X_2, X_3, \dots, X_n$.

So, the functional equation is written as

$$X = f(X_1, X_2, X_3, \dots, X_n)$$

2. Then the equation is expressed in terms of a constant with exponents like powers of a, b, c

Therefore, the equation is again written as

$$X = \phi(X_1^a, X_2^b, X_3^c, \dots, X_n^z)$$

Here $\phi = \text{constant}$ $a, b, c, \dots, z = \text{arbitrary power}$

3. The values of $a, b, c, \dots, z$ are determined with the help of dimensional homogeneity.

It means the power of the fundamental dimensions on both sides are compared to determine the values of exponents.

4. Finally, these exponents / power values are substituted in the functional equation and simplified to obtain the suitable form.

71) Find an expression for the drag force on smooth sphere of diameter D , moving with a uniform velocity V in a fluid density ρ and dynamic viscosity μ .

Sol:

Drag force F is a function of

- i) Diameter, D ii) Velocity, V iii) Density, ρ
iv) Viscosity, μ

$$F = K(D^a \cdot V^b \cdot \rho^c \cdot \mu^d) \rightarrow \text{① } K - \text{non-dimension constant.}$$

Substituting the dimensions on both sides,

$$MLT^{-2} = K[(L^a) \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (ML^{-1}T^{-1})^d]$$

Equating Powers of M , L and T on both sides,

Power of M , $\Rightarrow 1 = c + d$

Power of L , $\Rightarrow 1 = a + b - 3c - d$

Power of T , $\Rightarrow -2 = -b - d$

There are four unknowns (a, b, c, d) but equations are three. Hence it is not possible

$F = N = mg \rightarrow \frac{N}{s^2}$
 $\downarrow$
 MLT^{-2}
 $D = L \text{ or } m$
 $V = \frac{m}{s} = \frac{L}{T} = LT^{-1}$
 $\rho = \frac{m}{V} = \frac{M}{m^3} = ML^{-3}$

But three of them are known in terms of fourth variable which is most important.

Here viscosity is major role and hence a, b, c are expressed in terms of d

$$c = 1 - d$$
$$b = 2 - d$$

$$\therefore a = 1 - b + 3c + d = 1 - (2 - d) + 3(1 - d) + d$$

$$= 1 - 2 + d + 3 - 3d + d$$

$$= 2 - d$$

Substituting these values of a, b and c in eq (1)

$$F = K [D^{2-d} \cdot v^{2-d} \cdot \rho^{1-d} \cdot \mu^d]$$

$$= K D^2 v^2 \rho [D^{-d} \cdot v^{-d} \cdot \rho^{-d} \cdot \mu^d]$$

$$= K \rho D^2 v^2 \left[\frac{\mu^d}{D^d v^d \rho^d} \right]$$

$$= K \rho D^2 v^2 \left[\frac{\mu}{\rho v D} \right]^d$$

$$= K \rho D^2 v^2 \phi \left[\frac{\mu}{\rho v D} \right]$$

Surface tension $\sigma = MT^{-2}$

Speed $\rightarrow N = T^{-1}$ $N = \text{rpm} \Rightarrow \frac{r}{\text{min}} = \frac{1}{\text{min}} = \frac{1}{T} = T^{-1}$

Bulk modulus $K = ML^{-1}T^{-2}$

$\Delta P = \frac{N}{m^2} = \frac{ML^{-1}T^{-2}}{m^2}$

d, v, ρ

d, v, ρ

d, v, μ

d, v, μ

568

Given: ~~data~~

Solution:

velocity

3-81 Pg.

APOS

F is the function U, ρ, μ, D & N

(or)

The variables involved in analysis are F, U, ρ, μ, D & N

The dimensions of each variable are

Force $F = MLT^{-2}$

velocity, $U = LT^{-1}$

Density, $\rho = ML^{-3}$

Dynamic viscosity, $\mu = ML^{-1}T^{-1}$

Diameter, $D = L$

Speed, $N = T^{-1}$ $\text{rpm} \quad \frac{r}{m} = \frac{1}{s} = \frac{1}{T} = T^{-1}$

$h \leftarrow$ dimensionless
Parameter
so
 $M^0 L^0 T^0$

The functional Relationship can be written as

$F = f(U, \rho, \mu, D, N) \rightarrow \text{①}$

Again it can be written as

$f_1(F, U, \rho, \mu, D, N) = 0$

Total number of variables, $n = 6$

number of fundamental
variables } $m = 3$

$\therefore$ The number of π -terms $= n - m = 6 - 3 = 3$

So, the functional eqn. (π -terms)

$$f_1(\pi_1, \pi_2, \pi_3) = 0 \quad \text{--- (2)}$$

Each π -term has $m+1$ variables $= 3+1 = 4$

$$\pi_1 = D^{a_1} \cdot V^{b_1} \cdot e^{c_1} \cdot F$$

$$\pi_2 = D^{a_2} \cdot V^{b_2} \cdot e^{c_2} \cdot \mu$$

$$\pi_3 = D^{a_3} \cdot V^{b_3} \cdot e^{c_3} \cdot N$$

1st π term:

$$\pi_1 = D^{a_1} \times V^{b_1} \times e^{c_1} \times F$$

$$M^0 L^0 T^0 = L^{a_1} \times (LT^{-1})^{b_1} \times (ML^{-3})^{c_1} \times MLT^{-2}$$

Power of M , $0 = c_1 + 1 \Rightarrow \boxed{c_1 = -1}$

Power of L , $0 = a_1 + b_1 - 3c_1 + 1 \Rightarrow a_1 = -b_1 + 3c_1 - 1$

Power of T , $0 = -b_1 + 2 \Rightarrow \boxed{b_1 = 2}$

$\downarrow$

$$a_1 = -1 + 3(-1) - 1 = -5$$

Sub a_1, b_1, c_1 values in π_1

$$\pi_1 = D^{-5} \times V^2 \times e^{-1} \times F$$

$$\pi_1 = D^{-2} \times V^{-2} \times e^{-1} \times F$$

$$\boxed{\pi_1 = \frac{F}{D^2 V^2 e}}$$

2nd term:

$$\pi_2 = D^{a_2} \times U^{b_2} \times e^{c_2} \times \mu$$

$$M^0 L^0 T^0 = L^{a_2} \times (LT^{-1})^{b_2} \times (ML^{-3})^{c_2} \times ML^{-1} T^{-1}$$

Power of M, $0 = c_2 + 1 \Rightarrow \boxed{c_2 = -1}$

Power of L, $0 = a_2 + b_2 - 3c_2 - 1 \Rightarrow a_2 = -b_2 + 3c_2 + 1$

Power of T, $0 = -b_2 - 1 \Rightarrow \boxed{b_2 = -1}$

$$= -(-1) + (3(-1)) + 1$$

$$= 1 - 3 + 1 \Rightarrow 2 - 3$$

$$\boxed{a_2 = -1}$$

Substitute a_2, b_2, c_2 values in π_2 term.

$$\pi_2 = D^{-1} \times U^{-1} \times e^{-1} \times \mu$$

$$= D^{-1} \times U^{-1} \times e^{-1} \times \mu$$

$$\boxed{\pi_2 = \frac{\mu}{D U e}}$$

3rd term:

$$\pi_3 = D^{a_3} \times U^{b_3} \times e^{c_3} \times N$$

$$M^0 L^0 T^0 = L^{a_3} \times (LT^{-1})^{b_3} \times (ML^{-3})^{c_3} \times T^{-1}$$

Power of M, $0 = c_3 \Rightarrow \boxed{c_3 = 0}$

" L, $0 = a_3 + b_3 - 3c_3 \Rightarrow \boxed{a_3 = -b_3 + 3c_3}$

" T, $0 = -b_3 - 1 \Rightarrow \boxed{b_3 = -1}$

$$a_3 = -(-1) + 3(0)$$

$$\boxed{a_3 = 1}$$

Sub a_3, b_3, c_3 values in π_3 term.

$$\pi_3 = D^1 \times U^{-1} \times e^0 \times N$$

$$= D^1 \times U^{-1} \times e^0 \times N$$

$$\boxed{\pi_3 = \frac{DN}{U}} \Rightarrow \boxed{\frac{ND}{U} = \pi_3}$$

Sub the π_1, π_2, π_3 values in eqn (2)

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

$$f_1\left(\frac{F}{D^2 \nu^2 e}, \frac{\mu}{D \nu e}, \frac{ND}{U}\right) = 0$$

$$\frac{F}{D^2 \nu^2 e} = f\left(\frac{\mu}{D \nu e}, \frac{ND}{U}\right)$$

$$\frac{F}{D^2 \nu^2 e} = f\left(\frac{D \nu e}{\mu}, \frac{ND}{U}\right)$$

due to dimensionless parameter, $\frac{\mu}{D \nu e} = \frac{D \nu e}{\mu}$

Model (or) Similarity Laws

dynamic similarity is known as model (or) similarity laws.

- Reynolds model Law
- Froude " "
- Euler " "
- Model Law →
- Mach model Law

Reynolds model law:

is the ratio between the inertia force to the viscous force.

Reynolds model states that, the Reynolds number is same for both the model & prototype.

$$(Re)_{\text{model}} = (Re)_{\text{prototype}}$$

$$Re = \frac{\rho V L \text{ (or) } D}{\mu} \quad \text{con } x$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$\rho_m \rightarrow$ density of fluid in model

Time scale ratio $T_r = \frac{L_r}{V_r}$

discharge " " $Q_r = (\rho A V)_r$

Applications

- motion of air planes
- " " submarines
- Flow of incompressible fluid in closed pipes.
- Flow around structures.

Froude model law:

The Froude number is same for both model & prototype.

APP

- Flow of jet through orifice or nozzle.
- different mass densities of fluid flows over one another.

$$(F_r)_{\text{model}} = (F_r)_{\text{prototype}}$$

(Length scale ratio)

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

$$\Rightarrow \frac{V_m}{\sqrt{g_m}} = \frac{V_p}{\sqrt{g_p}} \quad \text{if } (g_m = g_p)$$

$$= \frac{V_p}{\sqrt{g_p}} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}$$

$$F_r = \frac{V}{\sqrt{g L}}$$

$$(F_r)_m = (F_r)_p \Rightarrow F_r = \frac{V}{\sqrt{gL}} \quad F_r \leftarrow \text{Force scale ratio.}$$

$$\frac{V_p}{V_m} = V_r \leftarrow \text{velocity scale ratio} \quad \frac{L_p}{L_m} = L_r \leftarrow \text{Length scale ratio}$$

$$V_r = \sqrt{L_r}$$

Time scale ratio (T_r)

$$T_m = \frac{L_m}{V_m} ; T_r = \sqrt{L_r}$$

$$T_p = \frac{L_p}{V_p}$$

$$Eu = \frac{V}{\sqrt{\rho \sigma}} \quad \rho_m = \rho_p \text{ for same fluid}$$

Euler model law $(Eu)_{\text{model}} = (Eu)_{\text{prototype}}$

Euler no. is same for both prototype & model.

App Phenomena: The gravity & surface tension forces are fully absent & the turbulence is fully developed with negligible viscous force.

Weber model law

is same for both model & prototype

$$\frac{V_m}{\sqrt{\rho_m}} = \frac{V_p}{\sqrt{\rho_p}} \left| \frac{V_m}{\sqrt{\rho_m \times L_m}} = \frac{V_p}{\sqrt{\rho_p \times L_p}} \right.$$

Sufficient criterion:

- 1) capillary rise in narrow passages
- 2) " waves in channels
- 3) " moment of water in sail
- e) very thin film of liquid flowing over a surface.

$$(We)_{\text{model}} = (We)_{\text{prototype}}$$

$$We = \frac{V_m}{\sqrt{\sigma \rho L}}$$

$$\frac{V_m}{\sqrt{\sigma_m L_m}} = \frac{V_p}{\sqrt{\sigma_p L_p}}$$

Types of models:

→ Undistorted models

→ Distorted "

① undistorted models

The model which is geometrically similar to its ^{type} prototype is known as undistorted model. In such models, The conditions of similitude are fully satisfied.

The Results obtained from the model are used to Predict the performance of the prototype easily.

② distorted models

A model which is not geometrically similar to its prototype but it maybe similar in appearance with its prototype. types

→ Geometrical distortion

→ material "

→ Distortion of hydraulic quantities!

Reasons of adopting distorted models:

→ To maintain accuracy.

→ To obtain suitable bed levels.

→ " " required roughness conditions.

Mach model law

is same for both model & prototype.

APP

- 1) Aerodynamic testing
- 2) Hydraulic model testing.

$$(M)_{\text{model}} = (M)_{\text{prototype}} \quad c_m \quad c_p$$

$$\frac{V_m}{\sqrt{K_m}} = \frac{V_p}{\sqrt{K_p}} \quad K_m = K_p \Rightarrow \text{elastic force.}$$

Classification of Models:

- undistorted models
- distorted models

undistorted

models ^{which} ~~with~~ are geometrically similar to their prototype.

(or) If the scale ratio for the linear dimensions of the model & its prototype is same.

Distorted models:

not geometrically similar to its prototype.

Adv

- 1) The cost of the model can be reduced
- 2) vertical dimensions of the model can be measured accurately.

Dimensionless Parameters:

Inertia force is always in a fluid flow.

Kinematic viscosity
 $\nu = \frac{\mu}{\rho}$

1) Reynolds number:

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho V D}{\mu} \quad \text{or} \quad \frac{D V}{\nu}$$

2) Froude number:

$$Fr = \sqrt{\frac{\text{Inertia force}}{\text{Gravity force}}} = \sqrt{\frac{V^2}{g L}}$$

3) Euler number

$$Eu = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}} = \frac{V}{\sqrt{\rho e}}$$

4) Weber number

$$We = \sqrt{\frac{\text{Inertia force}}{\text{Surface tension force}}} = \sqrt{\frac{\rho L V^2}{\sigma}}$$

5) Mach number:

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho v^2}{K}}$$

Application

- To check the dimensional homogeneity
- To reduce the dimensional variables from non-dimensional variables
- To derive the unknown parameter by using basic parameters (L.M.T)

Height scale ratio, $h_r = \frac{h_p}{h_m} = L_r$

Height of model, (H_m)

head scale ratio, $H_r = \frac{H_p}{H_m} = L_r$

$H_m = \text{Height of model}$

Discharge scale ratio, $Q_r = \frac{Q_p}{Q_m}$

Velocity scale ratio $V_r = \sqrt{(L_r)} = \frac{V_p}{V_m}$

Drag force, $F = ma \Rightarrow m = \rho L^3$ $\rho = \frac{m}{V} = m = \rho V \leftarrow m^3$
 $= \rho L^3$
 $a = \frac{V}{T} \rightarrow v = \frac{L}{T} = \frac{m}{s} = \frac{L}{T}$

Pressure drop scale ratio $P_r = \frac{P_p}{P_m}$

580 → problem: 12-17

①

Given: prototype (sub-marine) & its model.

For prototype, speed $V_p = 10 \text{ m/s}$

Fluid = Sea water

To find

① speed of air

② drag force ratio

Kinematic viscosities $V_p = 0.012 \text{ Stokes}$
 $= 0.012 \times 10^{-4} \text{ m}^2/\text{s}$

density

$\rho_p = 1030 \text{ kg/m}^3$

For model

Fluid = Air

$= 0.016 \text{ Stokes}$

Kinematic viscosity

$V_m = 0.016 \times 10^{-4} \text{ m}^2/\text{s}$

density,

$\rho_m = 1.24 \text{ kg/m}^3$

Length scale ratio $L_r = 30$.

Sol

viscous resistance so Reynold's number used.

Speed of air

$$\rho_p V_p \frac{L_p}{\mu_p} = \frac{\rho_m V_m L_m}{\mu_m}$$

$$\frac{V_p L_p}{\nu_p} = \frac{V_m L_m}{\nu_m}$$

$$V_m = \frac{\nu_m}{\nu_p} \times \frac{L_p}{L_m} \times V_p$$

$$= \frac{0.016 \times 10^{-4}}{0.012 \times 10^{-4}} \times 30 \times 10$$

$$V_m = 400 \text{ m/s}$$

~~$\rho_m \mu_p$~~

$$\nu = \frac{\mu}{\rho}$$

drag force [F]

$$F = m \times a \Rightarrow eL^3 \times \frac{V}{T} \Rightarrow eL^2 \times \frac{L}{T} \times V$$

$$F = eL^2 V^2$$

$$F_r = \frac{F_p}{F_m} = \frac{e_p L_p^2 V_p^2}{e_m L_m^2 V_m^2}$$

$$= \frac{e_p}{e_m} \times \left(\frac{L_p}{L_m} \right)^2 \times \left(\frac{V_p}{V_m} \right)^2$$

$$= \frac{1030}{1.24} \times (30)^2 \times \left(\frac{10}{400} \right)^2$$

$$\boxed{\frac{F_p}{F_m} = 467.22}$$

$$v = \frac{L}{T}$$

②

Unit - IV

Pumps

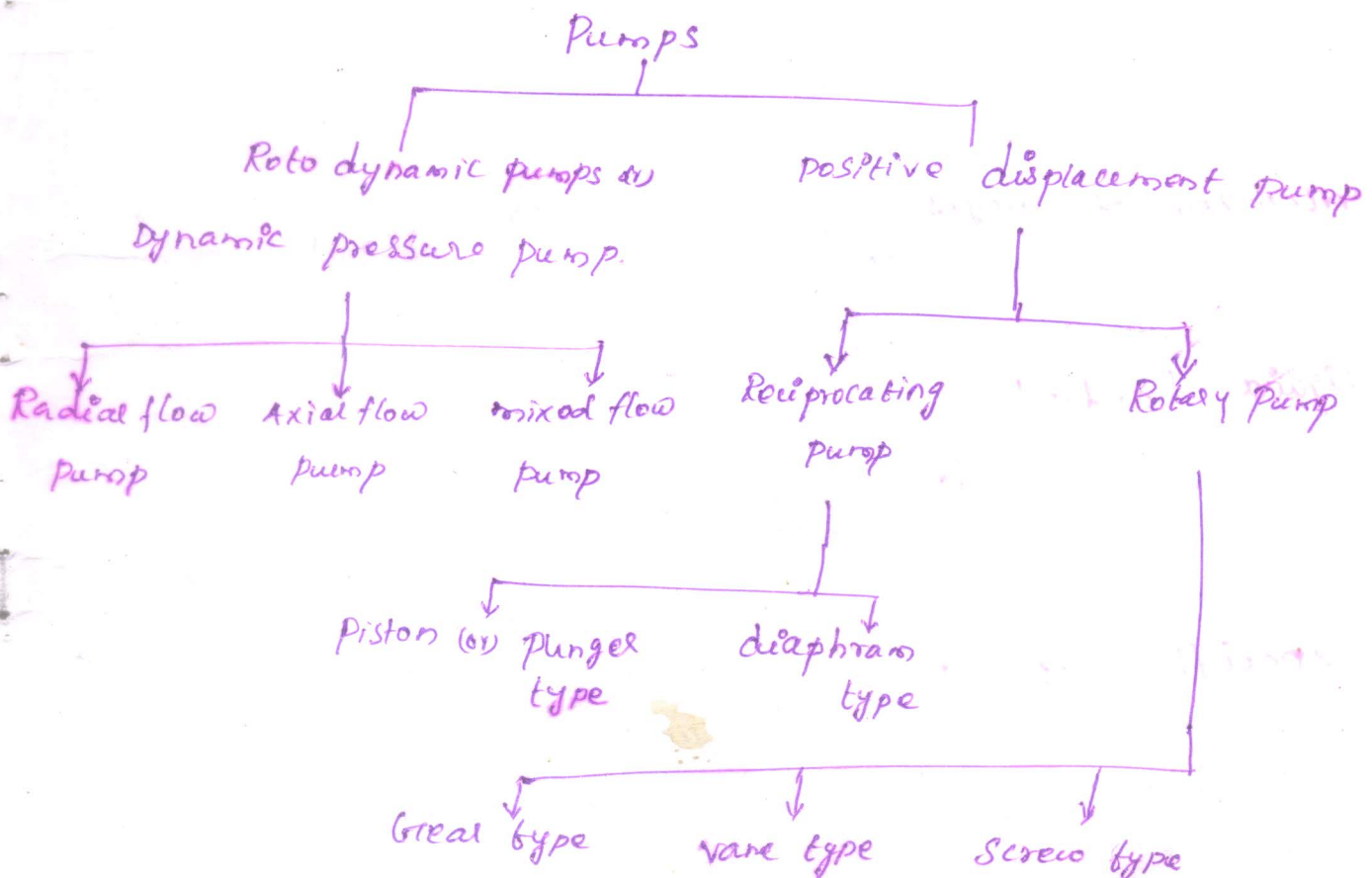
Pump:

Pump is a device which converts the mechanical energy supplied into hydraulic energy by lifting water to higher levels.

hydraulic energy refers to potential and kinetic energy of a liquid.

Lifting of water to higher levels is carried out by the various action of pumps such as centrifugal action, reciprocating action.

classification of pumps:



Centrifugal pumps:

centrifugal pumps are the device which converts mechanical energy into hydraulic energy by means of centrifugal force acting on the fluid.

cp are classified as roto dynamic type of pumps in which a dynamic pressure is developed to lift liquids from lower to a higher level.

classification of centrifugal pumps:

1. Shape and type of casing:
 - volute (or) spiral casing
 - Vortex (or) whirlpool casing
 - Volute casing with guide blades.
2. working head
 - low head (upto 15m)
 - medium head (15m to 40m)
 - High head (over 40m)
3. Number of stages
 - single stage cp
 - Multi stage cp
4. Liquid handled
 - closed impeller pump
 - Semi-closed impeller pump
 - open impeller pump.
5. Specified Speed:
 - low specific speed pumps
 - medium
 - High

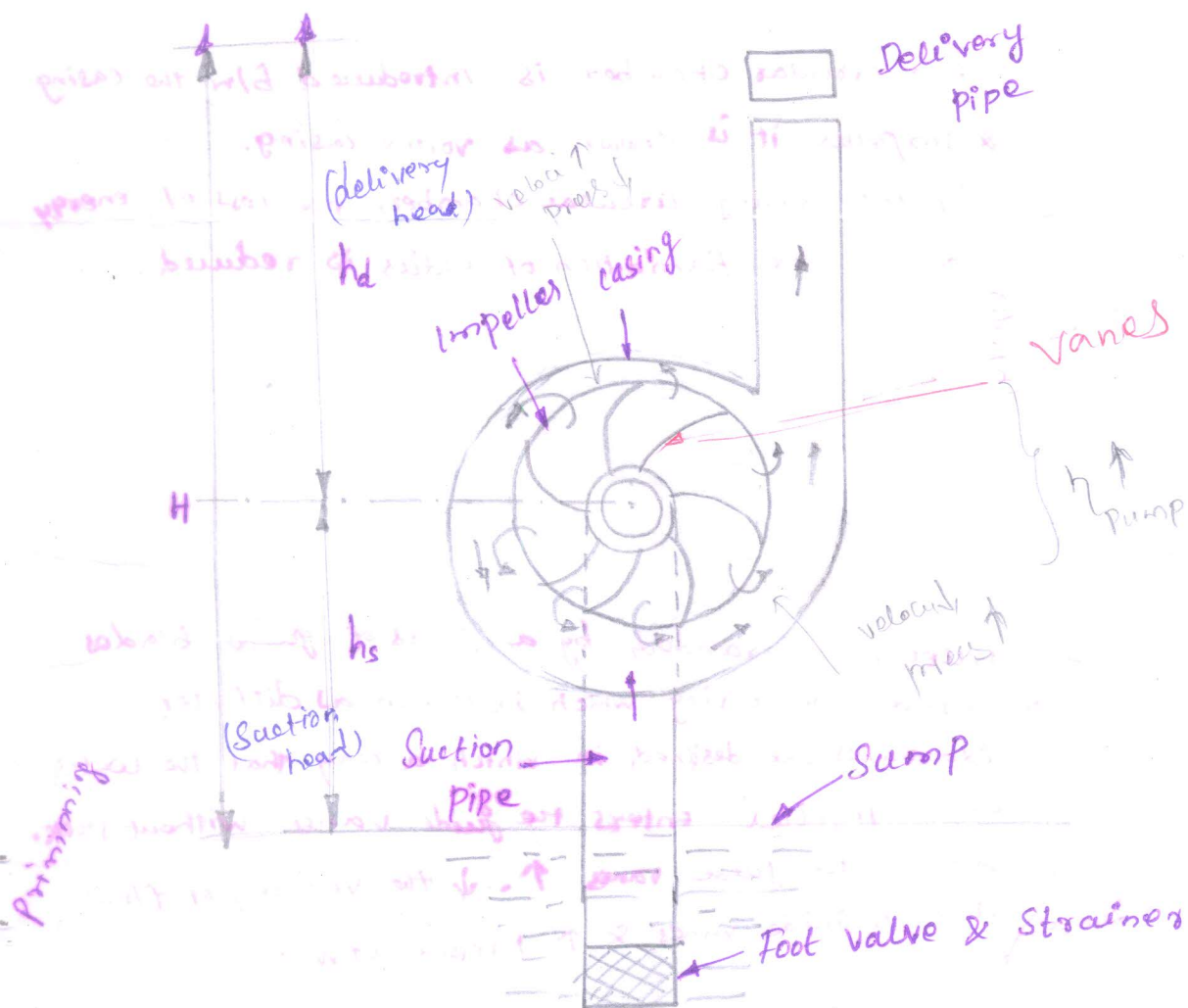
6. Number of entrances to the Impeller:

- Single Suction (or) single entry pump
- double Suction (or) double entry pump.

Main Components of a centrifugal pump:

A centrifugal pump consists of the following elements.

- Impeller
- casing
- Suction pipe, Strainer and foot valve.
- Delivery pipe & delivery valve.



Components of a centrifugal pump

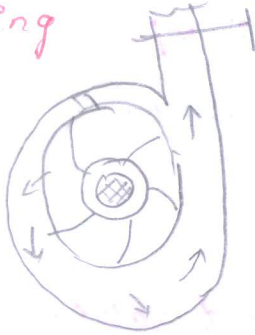
used to guide the water

(9)

Types of casing → kinetic energy convert to pressure energy

- Volute casing
- Vortex casing
- Volute casing with guide blades

Volute casing



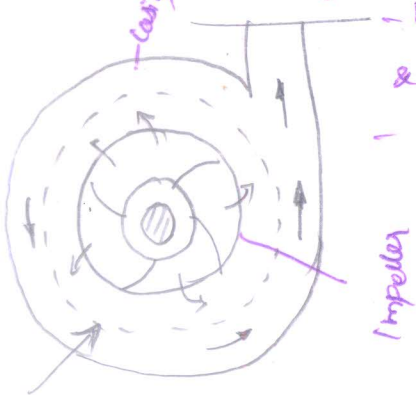
- Volute casing surrounds the impeller
- It is of spiral type in which area of flow increase gradually.

→ $\uparrow$ Area of flow $\downarrow$ Velocity $\uparrow$ Pressure

→ $h_{\text{pump}} \uparrow$

→ energy is lost due to the formation of eddies

Vortex casing

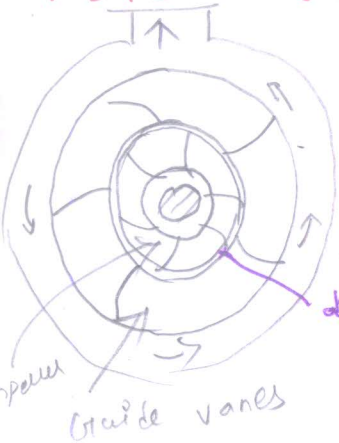


- If a circular chamber is introduced b/w the casing & impeller it is known as vortex casing.

- By introducing circular chamber, the loss of energy due to the formation of eddies is reduced.

Vortex casing

Volute casing with guide blades



- Impeller is surrounded by a series of guide blades mounted on a ring which is known as diffuser

- guide vanes are designed in such a way that the water from the impeller enters the guide vanes without shock.

- Area of the guide vanes $\uparrow \dots \downarrow$ the velocity of flow through guide vanes & $\uparrow$ pressure of water.

Impeller:

Rotating element consists of a vaned Rotor called impeller. the impeller mounted on a shaft which connected to the shaft of an electric motor.

The vanes of the impeller may be curved, cylindrical (or) have more complex surfaces.

But generally, series of backward curved vanes are equally spaced around the periphery of the shaft.

usually six to 12 vanes are located.

Types of Impeller:

- closed impeller
- Semi open type
- Open impeller

Various efficiencies:

- Hydraulic efficiency (η_h) (or) manometric
- Mechanical η (η_m)
- volumetric η (η_v)
- Overall η [η_o]

Hydraulic efficiency (η_h) (or) manometric

$$\eta_h = \frac{\text{Power developed by the runner}}{\text{Power supplied by the water jet}} = \frac{\text{Manometric Head}}{\text{Head imparted by Impeller to water}}$$

Mechanical η [η_m]

$$\eta_m = \frac{\text{Power available the turbine shaft}}{\text{Power developed by the turbine runner}} = \frac{\text{Power at impeller (P)}}{\text{Power at the shaft (or) mech power } P = W \cdot Q \cdot H_m ; W = \text{eg}}$$

$\frac{\text{shaft power}}{\text{water power}}$

η_m values lies b/w 97 to 99%

3. Volumetric η [η_v]

it is defined as the volume of water actually striking the buckets to the total water supplied by the jet

$$\eta_v = \frac{Q_a}{Q} \quad ; \quad \eta_v \text{ lies b/w } 0.97 \text{ to } 0.99$$

4. Overall η [η_o]

ratio of power available at the turbine shaft to the power available from the water jet

$$\eta_o = \frac{\text{Shaft Power}}{\text{Water Power}} = \frac{P}{WQH} \quad \eta_o \text{ values lies b/w } 0.85 \text{ to } 0.9$$

Mechanical Power = $\frac{2\pi NT}{60}$

N = speed in rpm
 T = Torque in N-m

Power of shaft

Power required to drive the pump, $P = \frac{WQH}{\eta_o}$

$W = wQ$

H_m = manometric head

W = weight of water

η_o = overall η

$$P = WQ(H_s + h_f)$$

w = weight density of water = $\frac{\rho g}{1000}$

impeller P = WQH

H_m = static head + Dynamic head

$$= H_m - \left[\frac{v_2^2 - v_1^2}{2g} \right]$$

$$H_m = \frac{v_{w2} \times v_2}{g}$$

$$v_2 = \sqrt{v_{f2}^2 + v_{w2}^2}$$

o/p Power = $P_{out/p} = WQH$

Brake power

$$\eta_o = \frac{\text{Water Power}}{\text{SP}} = \frac{\frac{WH_m}{1000}}{\text{SP}} = \frac{\frac{wQH_m}{1000}}{\text{SP}} = \frac{\frac{9.81 Q H_m}{1000}}{\text{SP}}$$

Formulas

Centrifugal Pumps:

Tangential velocity of impeller at inlet $[u_1]$

$$u_1 = \frac{\pi D_1 N}{60}$$

N = Speed of the impeller in rpm

D_1 = Diameter of impeller at inlet

u_1 = Tangential velocity of impeller at inlet

Tangential velocity of impeller at outlet

$$u_2 = \frac{\pi D_2 N}{60}$$

D_2 = dia of impeller at outlet

work done by the impeller on the water per sec / unit weight of water striking 1 second:

$$= \frac{1}{g} V_{w2} u_2$$

} work done / kg of water / second

weight of water $[W]$

$$W = \rho Q$$

Q = volume of water

or discharge

$$Q = A \times V_{\text{of flow}}$$

W - weight of water

$$= \pi D_2 B_2 \times V_{f2}$$

ρ = density of water = eg

B_1 & B_2 = width of impeller at inlet & outlet

$$= \pi D_1 B_1 \times V_{f1}$$

V_{f1} & V_{f2} = velocity of flow at inlet & outlet.

Manometric Efficiency $[h_{man}]$: (or) hydraulic

$$h = \frac{g \times H_m}{V_{w2} \times u_2}$$

H_m = manometric head

Mechanical h $[h_m]$

$$h_m = \frac{W}{g} \left[\frac{V_{w2} u_2}{1000} \right]$$

S.P. = Shaft Power

W = weight of water eg Q

V_{w2} = whirl velocity @ outlet

u_2 = velocity @ outlet

Tangential

Manometric Head [Hm]:

(Suction)

Hm = Total head at outlet of the pump - Total head at the inlet of the pump

net head ↑

$$H_m = \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o \right) - \left(\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + Z_i \right)$$

$\frac{P_o}{\rho g} = h_d$ ← Pressure head at outlet of the pump

$\frac{V_o^2}{2g}$ = Velocity head at outlet of the pump = $\frac{V_d^2}{2g}$

Z_o = vertical height of the outlet of the pump from datum line.

$$\frac{P_i}{\rho g} = h_s ; \quad \frac{V_i^2}{2g} = \frac{V_s^2}{2g} ; \quad Z_o = Z_s$$

$$V_d = \frac{Q}{A_d} ; \quad V_s = \frac{Q}{A_s}$$

A_s = Area of Suction pipe

A_d = Area of delivery pipe.

Overall efficiency [η_o]:

$$\eta_o = \frac{\frac{W H_m}{1000}}{S.P}$$

$$\eta_o = \eta_{mano} \times \eta_m$$

work done by impeller on water per second

$$\frac{W}{g} \times V_{w2} \times u_2$$

weight of water.

$$W = \rho g Q$$

ρ = weight density of water

Hm = head imparted by the impeller to liquid - loss of head in the pump

$$H_m = \frac{V_{w2} u_2}{g} - (h_{li} + h_{le})$$

h_{li} = Loss of head in the impeller
 h_{le} = " " in the casing

$$H_m = \frac{V_{w2} u_2}{g} \rightarrow \text{if there is no loss in the pump}$$

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

h_{fs} = frictional loss in the suction pipe

h_{fd} = " " " " delivery "

V_d = velocity of liquid in the delivery pipe

- 1) The internal & external ~~velocity~~ ^{diameter} of the impeller of a centrifugal pump are 200mm & 400mm respectively. The pump is running at 1200 rpm. the vane angles of the impeller at inlet and outlet are 20° & 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.

Given data:

Internal diameter of impeller	$D_1 = 200 \text{ mm} / 10^3 = 0.20 \text{ m}$
External " " "	$D_2 = 400 \text{ mm} / 10^3 = 0.40 \text{ m}$
speed	$N = 1200 \text{ rpm}$
Vane angle at inlet	$\theta = 20^\circ$
vane angle at outlet	$\phi = 30^\circ$
velocity of flow is constant	$V_{f1} = V_{f2}$
water enters radially means	$\alpha = 90^\circ \text{ \& } V_{w1} = 0$

To find:

- 1) work done by the impeller per unit weight of water.

Solution:

The work done by impeller per kg of water 1 second is

$$= \frac{1}{g} V_{w2} U_2$$

$$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60}$$

$$U_2 = 25.13 \text{ m/s}$$

$$\tan \phi = \frac{OP}{Adj}$$

$$\tan \phi = \frac{V_{f2}}{U_2 - V_{w2}}$$

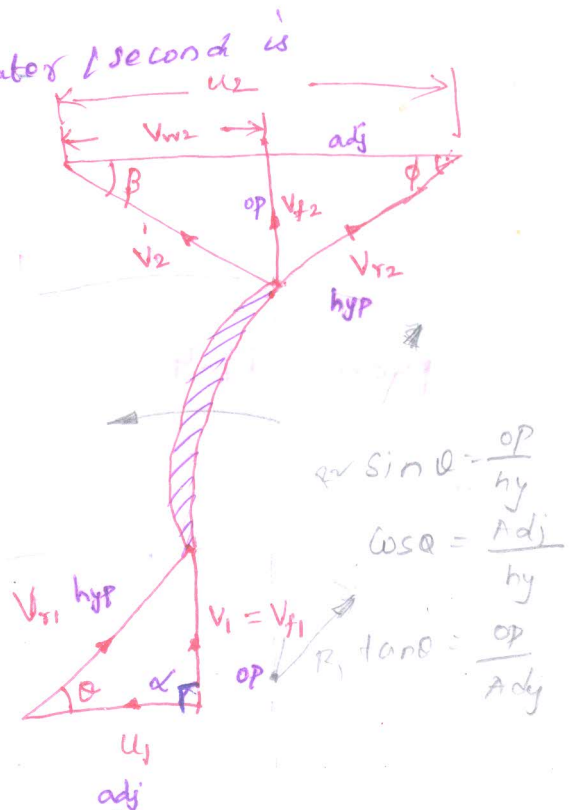
$$V_{f2} = V_{f1}$$

from outlet triangle
 $\tan \theta = \frac{OP}{Adj}$

$$\frac{OP}{Adj} = \frac{V_{f1}}{U_1}$$

$$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1200}{60}$$

$$U_1 = 12.56 \text{ m/s}$$



from intet triangle

$$\tan \phi = \frac{V_{f1}}{u_1} \Rightarrow \tan(20^\circ) = \frac{V_{f1}}{12.56}$$

$$\tan(20^\circ) \times 12.56 = V_{f1}$$

$$\boxed{V_{f1} = V_{f2} = 4.57 \text{ m/s}}$$

$$\tan \phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$\tan(30^\circ) = \frac{4.57}{25.13 - V_{w2}}$$

$$25.13 - V_{w2} = \frac{4.57}{\tan(30^\circ)}$$

$$25.13 - V_{w2} = 7.915$$

$$25.13 - 7.915 = V_{w2}$$

$$\boxed{V_{w2} = 17.215 \text{ m/s}}$$

$$\begin{aligned} -V_{w2} &= 7.915 - 25.13 \\ \sqrt{V_{w2}} &= 17.215 \\ \boxed{V_{w2} &= 17.215 \text{ m/s}} \end{aligned}$$

work done by impeller per kg of water / sec

$$= \frac{1}{g} V_{w2} \times u_2$$

$$= \frac{1}{9.81} \times 17.215 \times 25.13$$

$$= 44.1 \text{ Nm/kg}$$

2. Prob : 19.4

Solve in class by student

$$N = \text{kg} \frac{\text{m}}{\text{s}^2} \quad \left[\frac{\frac{\text{m}}{\text{s}} \times \frac{\text{m}}{\text{s}}}{\frac{\text{m}}{\text{s}^2}} = \text{N} \right]$$

$$\frac{\frac{\text{m}}{\text{s}} \times \frac{\text{m}}{\text{s}}}{\frac{\text{m}}{\text{s}^2}} = \frac{\frac{\text{m}^2}{\text{s}^2}}{\frac{\text{m}}{\text{s}^2}} = \frac{\text{m}^2}{\text{s}^2} \times \frac{\text{s}^2}{\text{m}} = \text{m}$$

3) A radial flow impeller has a diameter 25 cm and width 7.5 cm at exit. It delivers 120 litres of water per second against a head of 24 m at 1440 rpm. Assuming the vanes block the flow area by 5% and hydraulic efficiency of 0.8, estimate the vane angle at exit. Also calculate the torque exerted on the driving shaft if the mechanical efficiency is 95%.

Given data:

discharge $Q = 120 \text{ l/s} = \frac{120}{1000} = 0.120 \text{ m}^3/\text{s}$

diameter $D_2 = 25 \text{ cm} = \frac{25}{100} = 0.25 \text{ m}$

width $B_2 = 7.5 \text{ cm} = \frac{7.5}{100} = 0.075 \text{ m}$

Head $H_m = 24 \text{ m}$

Speed $N = 1440 \text{ rpm}$

hydraulic or manometric efficiency $\eta_{h/m} = 0.8$

Mechanical efficiency $\eta_m = 95\% = \frac{95}{100} = 0.95$

Solution:

$$\tan \phi = \frac{\text{opp}}{\text{adj}} = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1440}{60}$$

$$u_2 = 18.85 \text{ m/s}$$

Outlet of discharge

$$Q = 0.95 \pi D_2 B_2 \times V_{f2}$$

$$0.120 = 0.95 \times \pi \times 0.25 \times 0.075 \times V_{f2}$$

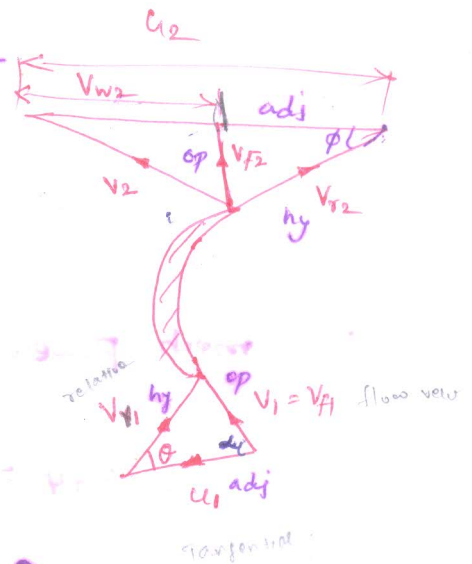
$$V_{f2} = 1.787 \text{ m/s}$$

$$\tan \phi = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{1.787}{18.85 - 15.61}$$

To find

① vane angle at exit $\phi = ?$

② Torque = ?



(only 95% of flow will occur at outlet due to 5% vane block)

$$\eta_{h/m} = \frac{g H_m}{V_{w2} u_2} = 0.8 = \frac{9.81 \times 24}{V_{w2} \times 18.85}$$

$$V_{w2} = 15.61 \text{ m/s}$$

Mechanical power $\left. \begin{matrix} \text{Power} \\ \text{Shaft of shaft} \end{matrix} \right\} = \frac{2\pi NT}{60}$

Mechanical efficiency, $\eta_{\text{mech}} = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$

$0.95 = \frac{23544}{\text{Power at the shaft}}$

Power at the shaft $\left. \begin{matrix} \text{(a)} \\ \text{mech power} \end{matrix} \right\} = \frac{23544}{0.95}$
 $= 24780 \text{ W}$

Shaft power = 2478 W

Power at the impeller, $P = \rho Q H_m$ $\rho = \text{eg}$

$= \rho g Q H_m$

$= 1000 \times 9.81 \times 0.12 \times 24$

$P = 23544 \text{ W}$

$\rightarrow \text{mech power} = \frac{2\pi NT}{60}$

$\frac{2478}{23544} = \frac{2\pi \times 1440 \times T}{60}$

$\frac{2478}{23544} \times 60 = 2\pi \times 1440 \times T$

$\frac{2478 \times 60}{2\pi \times 1440} = T$

$T = 164.33 \text{ N-m}$

$\frac{2478 \times 60}{2\pi \times 1440} = T$

prob 9.9

5

4) Find the Power required to drive a centrifugal pump which delivers $0.04 \text{ m}^3/\text{s}$ of water to a height ^{h_d} of 20 m through a 15 cm dia pipe $\times 100 \text{ m}$ Long. The overall efficiency of the pump is 70% & coefficient of friction is 0.15 in the formula $h_f = \frac{f L V^2}{2 g d}$

Given data:

$$Q = 0.04 \text{ m}^3/\text{s}$$

$$H = 20 \text{ m}$$

$$D = 15 \text{ cm} = \frac{15}{100} = 0.15 \text{ m}$$

$$L = 100 \text{ m}$$

$$\eta_o = 70\% = \frac{70}{100} = 0.70$$

$$f = 0.15$$

To find

required to drive a pump
Power (P)

$$h_f = h_{fs} + h_{fd}$$

$$h = h_s + h_d$$

Solution:

$$\text{Power required to drive a pump (P)} = \frac{\rho Q H_m}{\eta_o}$$

$$H_m = h_s + h_{fd} + \frac{V_d^2}{2g}$$

$$\text{Frictional head } h_{fd} = \frac{f L V^2}{2 g d}$$

$$= \frac{0.15 \times 100 \times \left(\frac{Q}{A}\right)^2}{2 \times 9.81 \times 0.15}$$

~~QZAY~~

$$h_{fd} = 26.11 \text{ m}$$

velocity of water in pipe

$$V_d = \frac{Q}{A} = \frac{0.04}{\frac{\pi}{4} (0.15)^2} = 2.26 \text{ m/s}$$

$$V_d = 2.26 \text{ m/s}$$

$$H_m = h_s + h_{fd} + \frac{V_d^2}{2g} = 20 + 26.11 + \frac{(2.26)^2}{2 \times 9.81}$$

$$H_m = 46.37 \text{ m}$$

$$Q = A \times V$$

$$V = \frac{Q}{A}$$

$$P = \frac{\rho Q H_m}{\eta_o}$$

$$H_m =$$

$$P = \frac{W Q H_m}{\eta_o} = \frac{\rho g Q H_m}{\eta_o} = \frac{1000 \times 9.81 \times 0.04 \times 46.37}{0.7}$$

$$P = \frac{25993.697}{10^3} \text{ W}$$

$$P = 25.993 \text{ kW}$$

(5) A centrifugal pump having outer diameter equal to 2 times the inner diameter and running at 1200 rpm works against a total head of 75m. The velocity of flow through the impeller is constant & equal to 3 m/s. The vanes are set back at an angle of 30° at outlet. If the outer diameter of the impeller is 600mm and width at outlet is 50mm. determine.

① vane angle at inlet (θ)

2. work done / second by impeller (W)

3. manometric efficiency. (η_{mano})

Given data:

$$D_2 = 2 D_1 = 0.6 = 2 D_1$$

$$\text{Speed, } N = 1200 \text{ rpm} \quad \boxed{D_1 = 0.3 \text{ m}}$$

$$\text{Head, } H_m = 75 \text{ m}$$

$$\text{Velocity of flow, } V_{f1} = V_{f2} = 3 \text{ m/s}$$

$$\text{Vane angle at outlet } \phi = 30^\circ$$

$$\text{Outer dia } D_2 = 600 \text{ mm} = 0.6 \text{ m}$$

$$\text{outlet width } B_2 = 50 \text{ mm} = 0.05 \text{ m}$$

Solution:

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{u_1}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 1200}{60}$$

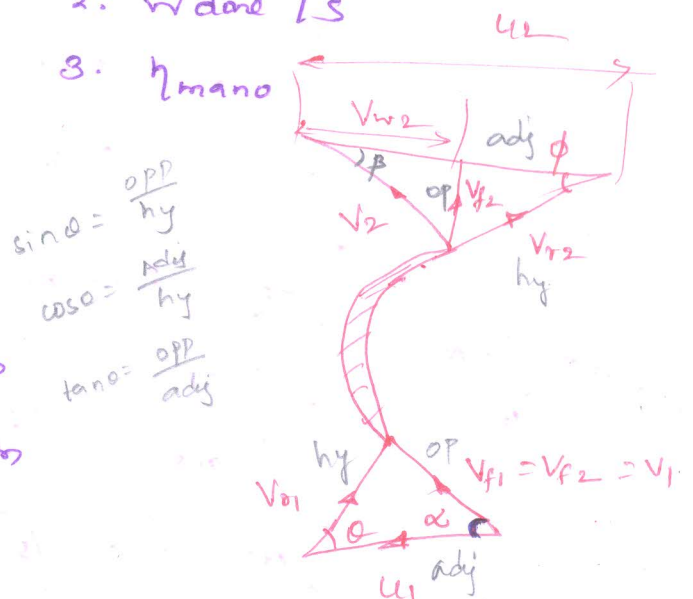
$$\boxed{u_1 = 18.849 \text{ m/s}}$$

To find

① vane angle @ inlet (θ)

2. work done / s

3. η_{mano}



$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{u_1} = \frac{3}{18.849}$$

$$\theta = \tan^{-1} \left(\frac{3}{18.849} \right) \leftarrow 0.15^\circ$$

$$\boxed{\theta = 9^\circ}$$

work done / second ~~the~~ by impeller

$$= \frac{W}{g} \times V_{w2} \times u_2$$

$$= \frac{\rho g Q}{g} \times V_{w2} \times u_2$$

W - weight of water (eg) $\left(\frac{W}{g} = \rho Q \right)$

ρ - weight density of water (eg)

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 1200}{60}$$

$$\boxed{u_2 = 37.7 \text{ m/s}}$$

$$\tan \phi = \frac{\text{opp}}{\text{adj}} = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{3}{37.7 - V_{w2}}$$

$$\tan(30^\circ) = \frac{3}{37.7 - V_{w2}} \Rightarrow \frac{3}{\tan(30^\circ)} = 37.7 - V_{w2}$$

$$5.196 = 37.7 - V_{w2}$$

$$\boxed{V_{w2} = 32.5 \text{ m/s}}$$

$$Q = \pi D_2 B_2 \times V_{f2} = \pi \times 0.6 \times 0.05 \times 3$$

$$Q = 0.28274 \text{ m}^3/\text{s}$$

$$\boxed{Q = 0.28274 \text{ m}^3/\text{s}}$$

$$= \frac{1000 \times 9.81 \times 0.28274}{9.81} \times 32.5 \times 37.7$$

$$W/s = 1225250 \text{ } \underline{346427.185 \text{ N m/s}}$$

$$1000/\text{m}^3 \times \frac{1}{\text{s}} \times \frac{1}{\text{s}} \times \frac{1}{\text{s}} \times \frac{1}{\text{s}} \times \frac{1}{\text{s}}$$

Manometric efficiency

$$h_{\text{mano}} = \frac{g H_m}{V_{w2} u_2} = \frac{9.81 \times 75}{32.5 \times 37.7}$$

$$h_{\text{mano}} = 0.60$$

$$\boxed{h_{\text{mano}} = 60\%}$$

$$\frac{\text{kg}}{\text{m}^3} \times \frac{1}{\text{s}} \times \frac{1}{\text{s}} \times \frac{1}{\text{s}} \times \frac{1}{\text{s}} \times \frac{1}{\text{s}}$$

8)
b) A centrifugal pump has 30 cm & 60 cm dia's at inlet & outlet. The inlet & outlet vane angles are 30° & 45° respectively. Water enters at a velocity of 2.5 m/s radially. Find the speed of impeller in rpm & power of the pump. If the flow is $0.2 \text{ m}^3/\text{s}$.

Given data:

$$D_1 = 30 \text{ cm} = \frac{30}{100} = 0.3 \text{ m}$$

$$D_2 = 60 \text{ cm} = \frac{60}{100} = 0.6 \text{ m}$$

$$\theta = 30^\circ$$

$$\phi = 45^\circ$$

$$V_{f1} = 2.5 \text{ m/s}$$

$$Q = 0.2 \text{ m}^3/\text{s}$$

Solution:

$$u_1 = \frac{\pi D_1 N}{60}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{u_1}$$

$$\tan(30^\circ) = \frac{V_{f1}}{u_1} = \tan(30^\circ) = \frac{2.5}{u_1}$$

$$u_1 = \frac{2.5}{\tan(30^\circ)} \Rightarrow \boxed{u_1 = 4.33 \text{ m/s}}$$

$$u_1 = \frac{\pi D_1 N}{60} \Rightarrow 4.33 = \frac{\pi \times 0.3 \times N}{60}$$

$$4.33 \times 60 = \pi \times 0.3 \times N$$

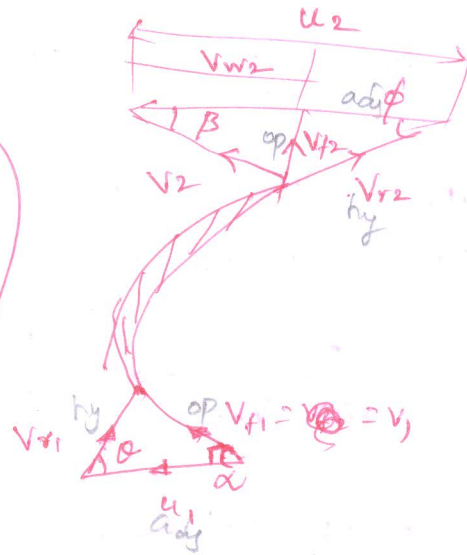
$$\frac{4.33 \times 60}{\pi \times 0.3} = N$$

$$\boxed{N = 275.66 \text{ rpm}}$$

To find

1) speed (N) in rpm

2) Power (P)



② power of the pump

⑨

$P =$

$(11.48 \text{ exp } 8) \text{ kW}$

6) A centrifugal pump impeller runs at 800 rpm and has outlet vane angle of 60° . The velocity of flow is 2.5 m/s throughout & dia of the impeller at exit is twice that at inlet. If the manometric head is 20 m & the manometric η is 75% determine the dia of the impeller at the inlet & inlet vane angle. (10)

Given data:

$D_2 = 2D_1$

speed $N = 800 \text{ rpm}$

velocity of flow $V_{f1} = V_{f2} = 2.5 \text{ m/s}$

vane angle at outlet $\phi = 60^\circ$

head $H_m = 20 \text{ m}$

$\eta_{\text{mano}} = 75\% = 0.75$

Sol

Tangential velocity @ inlet

$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi D_1 \times 800}{60}$

as last

②

$u_1 = 41.89 D_1 \text{ m/s}$

Tangential velocity @ outlet

$D_2 = 2D_1$

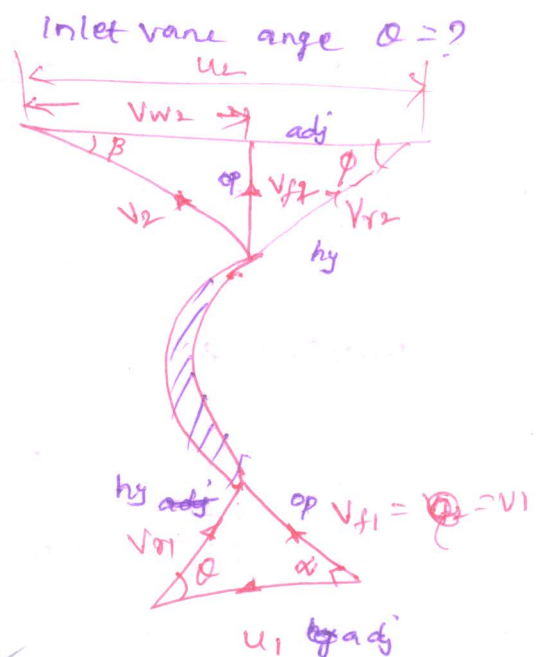
$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi 2D_2 \times 800}{60} = 41.8879 D_2$

②

$= 41.8879 \times 2 \times D_1$

$u_2 = 83.78 D_1 \text{ m/s}$

$= 83.7758 D_1 \text{ m/s}$



Manometric efficiency:

$$\eta_{\text{mano}} = \frac{g H_m}{V_{w2} u_2} \Rightarrow 0.75 = \frac{9.81 \times 20}{V_{w2} \times 83.78 D_1}$$

$$V_{w2} = \frac{3.1225}{D_1}$$

From outlet velocity & angle,

tan $\phi = \frac{V_{f2}}{u_2 - V_{w2}} \Rightarrow \tan(60^\circ) = \frac{2.5}{83.78 D_1 - \frac{3.1225}{D_1}}$

$$\frac{2.5 D_1}{83.78 D_1 \times 3.1225} \left[\frac{2.5}{\tan 60^\circ} = 83.78 D_1 - \frac{3.1225}{D_1} \right]$$

$$1.44 = 83.78 D_1 - \frac{3.1225}{D_1}$$

$$1.44 = \frac{83.78 D_1^2 - 3.1225}{D_1}$$

$$1.44 D_1 = 83.78 D_1^2 - 3.1225$$

$$83.78 D_1^2 - 3.1225 - 1.44 D_1 = 0$$

$$83.78 D_1^2 - 1.44 D_1 - 3.1225 = 0$$

a b c

$$D_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1.44) \pm \sqrt{(-1.44)^2 - (4 \times 83.78 \times -3.1225)}}{2 \times 83.78}$$

$$D_1 = \frac{1.44 \pm \sqrt{2.0736 + 1046.4122}}{2 \times 83.78}$$

Quadratic equation

$$D_1 = \frac{1.44 \pm \sqrt{1048.48}}{167.56}$$

$$= \frac{1049.92}{167.56}$$

(11)

$$D_1 = 0.2018 \text{ m}$$

dia of impeller @ entry $D_1 = 0.2018 \text{ m}$

" " @ exit $D_2 = 2D_1 = 2 \times 0.2018 \text{ m}$

$$D_2 = 0.4036 \text{ m}$$

② Inlet vane angle (θ) = ?

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{u_1}$$

$$\tan \theta = \frac{2.5}{8.453}$$

$$\tan \theta = 0.2957$$

$$\theta = \tan^{-1}(0.2957)$$

$$\theta = 16.475'$$

$$u_1 = 41.89 \times D_1$$

$$= 41.89 \times 0.2018$$

$$u_1 = 8.453 \text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$= \frac{\pi \times D_1 \times 800}{60}$$

$$u_1 = 41.89 D_1$$

Specific Speed for a Pump (N_s)

$$N_s = \frac{N \sqrt{Q}}{H_m^{3/4}} \quad \checkmark$$

defined as the speed of a geometrically similar pump which will deliver unit quantity (1 litre of liquid / sec) against a unit head (1 meter)

If the power required drive the pump is P ,

$$N_s = \frac{N \sqrt{P}}{H_m^{5/4}}$$

diff types of centrifugal pumps with the corresponding ranges of specific speeds:

S. no.	Types of impeller	Specific speed N_s
1.	Slow speed radial flow	10-30
2.	Medium speed radial flow	30-50
3.	High speed radial flow	50-80
4.	Mixed flow	80-160
5.	Axial flow	160-500

$$\text{Number of pumps} = \frac{H_{\text{total}}}{H_{\text{mano}}}$$

$$\text{overall } \eta = \frac{\text{Power o/p}}{\text{Power I/P (s.p.)}}$$

Power output of the pump = $W \cdot Q \cdot H_{\text{total}}$

total head delivered = $H_{\text{total}} = n \times H_m$

7) Find the number of pumps required and their arrangement to take water from a deep well under a total head of 89 m. All the pumps are identical and are running at 800 rpm. The specific speed of each pump is given as 25 while the rated capacity of each pump is $0.16 \text{ m}^3/\text{s}$.

Given data:

To find

Total head, $H_{\text{total}} = 89 \text{ m}$

number of pumps

Speed, $N = 800 \text{ rpm}$

Specific Speed, $N_s = 25$

Discharge, $Q = 0.16 \text{ m}^3/\text{s}$

Solution:

$$\text{Number of pumps} = \frac{H_{\text{total}}}{H_{\text{mano}} \text{ or } H_m}$$

$$\text{Specific speed } N_s = \frac{N \sqrt{Q}}{(H_m)^{3/4}}$$

$$25 = \frac{800 \times \sqrt{0.16}}{(H_m)^{3/4}}$$

$$(H_m)^{3/4} = \frac{800 \times \sqrt{0.16}}{25}$$

$$H_m = (12.8)^{4/3}$$

$$H_m = 29.94 \text{ m}$$

$$\text{Number of pumps} = \frac{H_{\text{total}}}{H_{\text{mano}}} = \frac{89}{29.94} = 2.97 \approx 3$$

3 pumps required to take water from a deep well

8) The head-discharge characteristics of a centrifugal pump is given below

discharge (lit/sec)	0	10	20	30	40	50
Head (m)	25.3	25.5	24.5	22.2	18.7	12.0

The pump delivers fresh water through a 500m long, 15cm diameter pipe line having friction coefficient of $f = 0.025$, the static (suction) lift is 15m. neglecting minor losses in the pipe flow,

find (i) The discharge of the pump under the above conditions

(ii) driving power of the pump motor. Assume a pump η of 72%

Given data

$$H_s = 15 \text{ m} = H_s$$

$$D = 15 \text{ cm} = \frac{15}{100} = 0.15 \text{ m}$$

$$L = 500 \text{ m}$$

$$f = 0.025$$

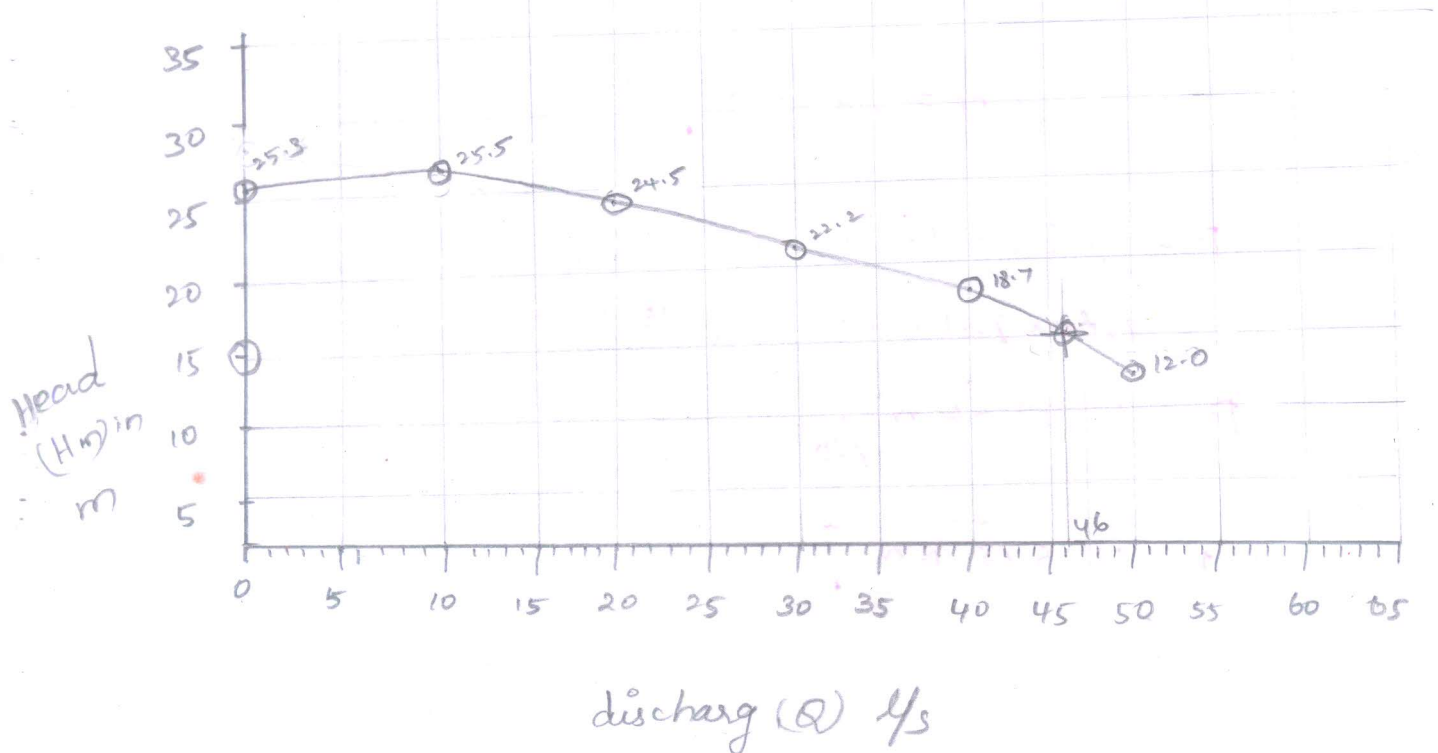
$$\eta = 72\%$$

To find

① discharge

② power ^{driving} of the pump motor.

Solution: First, the graph is drawn b/w head & discharge.



First, the graph

Corresponding to head, $H_s = 15\text{ m}$, read discharge as
 $= 46\text{ lit/s}$

Power required to run the pump rotor,

$$P = \frac{\rho Q (H_s + h_f + \frac{v^2}{2g})}{1000} \text{ W}$$
$$= \rho Q (H_s + h_f + \frac{v^2}{2g})$$

$$h_f = \frac{4fLv^2}{2gd}$$

$$Q = A \times v \rightarrow A = \frac{\pi D^2}{4} = \frac{\pi}{4} \times (0.15)^2$$

$$A = 0.0177\text{ m}^2$$

$$v = \frac{Q}{A} \Rightarrow \frac{46 \times 10^{-3}}{0.0177}$$

$$v = 2.6\text{ m/s}$$

$$h_f = \frac{4fLv^2}{2gd} = \frac{4 \times 0.025 \times 500 \times (2.6)^2}{2 \times 9.81 \times 0.15}$$

$$h_f = 114.85\text{ m}$$

$$P = \rho Q (H_s + h_f + \frac{v^2}{2g})$$
$$= 1000 \times 9.81 \times 46 \times 10^{-3} [15 + 114.85 + 0.3445]$$

$$P = 58573.55\text{ W} / 10^3$$

$$P = 58.573\text{ kW}$$

$$= 81.35 \times 10^3\text{ W}$$

9) A centrifugal pump with backward-curved blades has the following measured performance when tested with water @ 20°C

discharge (lit/min)	0	1800	3600	5400	7200	9000	10800	
Head (cm)	37	35	33	31	28	25	19	
power (kW)	22	27	30	33	35	36	34	

Estimate the best efficiency point and the maximum efficiency. Also, estimate the most efficiency flow rate, and the resulting head and brake power, if the diameter is doubled & the rotating rotation speed is increased by 50%.

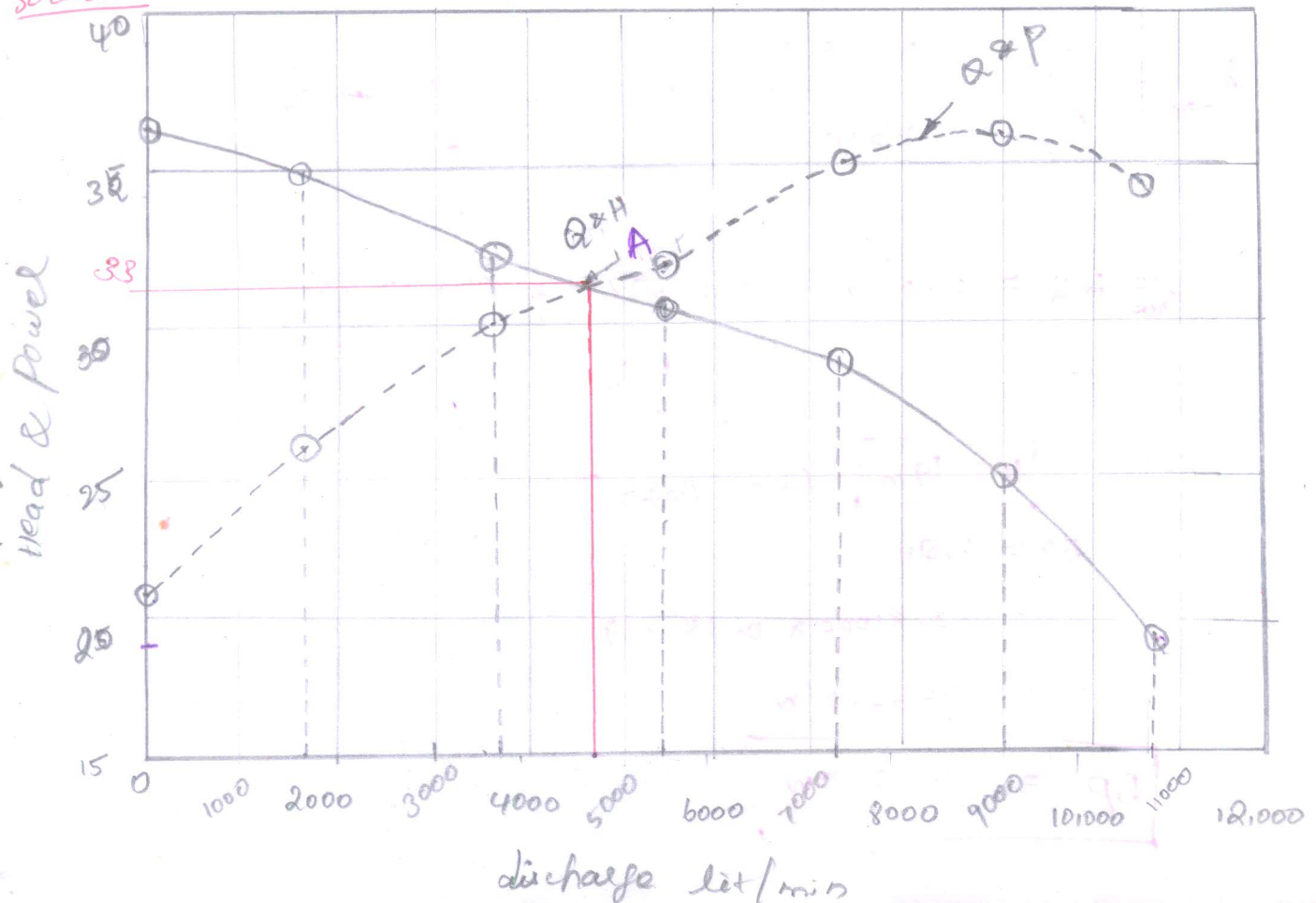
To find:

- 1) best η point
- 2) Max η
- 3) most η flow rate
- 4) Resulting head
- 5) Brake power.

① discharge & Head

$Q \propto P$

Solution



- ① The meeting point A is the best efficiency point

At the point, discharge, $Q = 5400 \text{ lit/min}$

$$\textcircled{1} \quad = 5400/1000 = 5.4 \text{ m}^3/\text{min}$$

$$\boxed{Q = 0.09 \text{ m}^3/\text{s}}$$

head, $H = 33 \text{ m}$

power, $P_{\text{input}} = 33 \text{ kW}$

} from graph
(16 pg. no.)
(front page) ←

Power o/p @ the best h point

$$P_{\text{output}} = W \overset{\text{eg}}{Q} H$$

$$= 9.81 \times 1000 \times 0.09 \times 33$$

$$\boxed{P_{\text{opp}} = 29135.7 \text{ W}}$$

② Maxi efficiency:

$$\text{Max eff. } \eta_{\text{max}} = \frac{P_{\text{opp}}}{P_{\text{i/p}}} = \frac{29135.7}{33 \times 10^3} = 0.8829 \quad \boxed{= 88.29 \%}$$

③ most h flow rate:

$$Q_{\text{new}} = 2Q = 2 \times 0.09 = 0.18 \text{ m}^3/\text{s} = 0.18 \times 10^3 \times 60$$

$$\boxed{Q_{\text{new}} = 10,800 \text{ lit/min}}$$

④ Resulting head $\boxed{H_R = 19 \text{ m}}$ from ~~graph~~ Table (or) graph

⑤ Brake power: $BP = WQH_R$

$$= 9.81 \times 1000 \times 0.18 \times 19$$

$$= 33550.2 \text{ W}$$

$$\boxed{BP = 33.55 \text{ kW}}$$

Cavitations in hydraulic machine

cavitation is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapour pressure & the sudden collapsing of these vapour bubbles in a region of high pressure.

Effect of cavitation:

- sudden drop in head h & power delivered to the fluid.
- noise & vibrations produced by the collapse of vapour bubbles.

Reciprocating Pump

Reciprocating pump is a positive displacement pump. it operates on the principle of actual displacement or pushing of liquid by a piston or plunger that reciprocates in a closely fitting cylinder.

In pump mechanical energy convert into hydraulic energy.

Application:

- light oil pumping
- Feeding small boiler condensate return.
- Pneumatic pressure system.

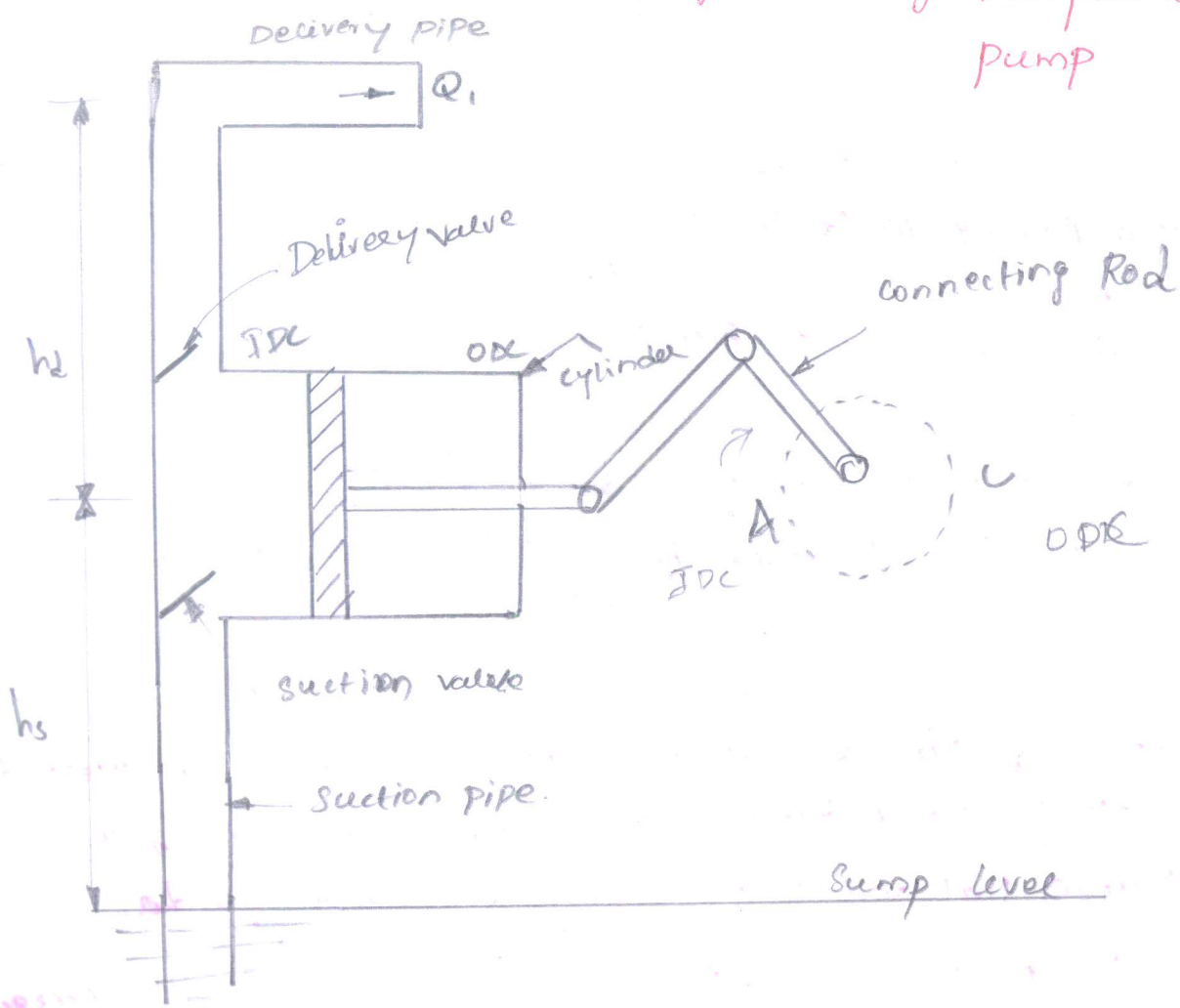
classification of Reciprocating pump:

The reciprocating pumps are classified as follows:

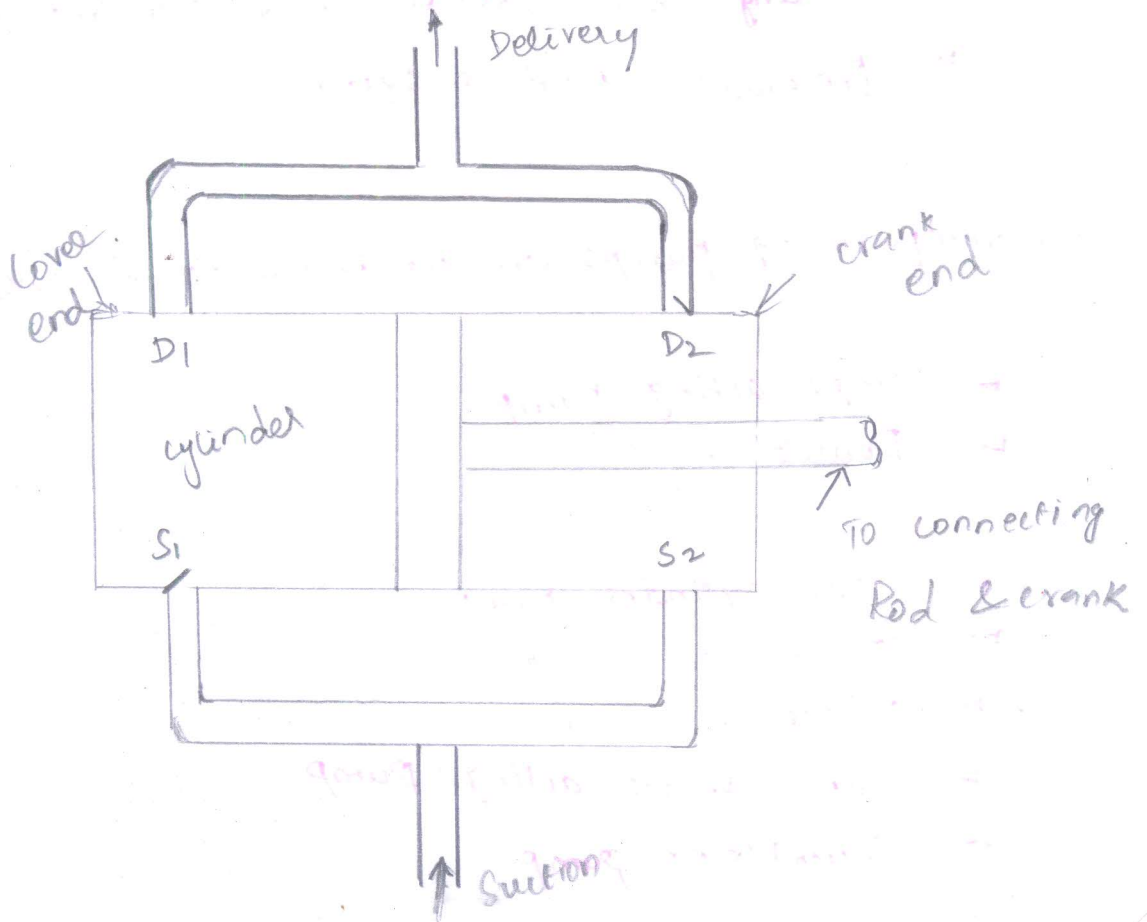
1. According to the fluid being in contact with piston (or) plunger
 - single acting pump
 - Double " "
2. According to the number of cylinders provided
 - single cylinder pump
 - double " "
 - Triple " "
 - duplex double acting pump
 - Quantiplex pump

Single acting Reciprocating Pump

(19)



Double acting Reciprocating Pump:



Reciprocating Pumps

single acting pump

$$A = \pi/4 D^2$$

$D \rightarrow$ diameter of cylinder

$r \rightarrow$ Radius of crank

$N \rightarrow$ speed of the crank in rpm

$L \rightarrow$ length of the stroke ($2r$)

$h_s =$ Height of the axis of the cylinder from water surface in Sump

$h_d =$ Height of the delivery outlet above the cylinder axis

$w =$ weight density of liquid
 $\hookrightarrow eg$

Discharge of the Pump per second

(theoretical dis)

$$Q = \frac{A L N}{60} \text{ m}^3/\text{s}$$

weight of water delivered per second

$$W = w Q$$

$$W = \frac{w A L N}{60}$$

$w = eg$

work done by Reciprocating pump.

work done / sec = $\frac{eg A L N}{60} \times (h_s + h_d) + h_{fs} + h_{fd}$ for single acting pump

Power (P) = $\frac{eg A L N \times (h_s + h_d) + h_{fs} + h_{fd}}{60 \times 1000} \text{ kW}$ (or) $\frac{w A L N}{60} \times (h_s + h_d) \text{ in W}$ (single act)

work done / 1000

Double acting Pump

area of piston

$$A = \frac{\pi}{4} (D^2)$$

$$Q_{th} = \frac{2 A L N}{60}$$

for double acting pump

work done = $2 eg \frac{A L N}{60} \times (h_s + h_d) + h_{fs} + h_{fd}$

Power (P) = $\frac{2 eg A L N \times (h_s + h_d)}{60,000} \text{ kW}$

$\frac{2 w A L N}{60} \times (h_s + h_d) \text{ in W}$

slip of a Reciprocating pump:

(2)(2)

$$\text{slip} = Q_{th} - Q_{act} \quad m^3/s$$

Q_{th} = theoretical discharge

Q_{act} = actual discharge

C_d = coefficient of discharge

$$\% \text{ slip} = (1 - C_d) \times 100$$

$$C_d = \frac{Q_{act}}{Q_{th}}$$

no unit

$$\Rightarrow \text{or } \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$\Rightarrow \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = 1 - \frac{Q_{act}}{Q_{th}} \times 100$$

$$\text{Theoretical Power } (P_T) = \frac{\rho g Q_{th} H}{1000}$$

Acceleration head at the beginning of suction ~~to~~ stroke (on) pipe

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos \theta$$

for Suction pipe

ω = angular velocity
 $\omega = \frac{2\pi N}{60}$

A → Area of piston, a_s → area of suction pipe

Acceleration head at the beginning of delivery stroke (on) pipe

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_s} \omega^2 r \cos \theta$$

for delivery pipe $\omega = \frac{2\pi N}{60}$

$$\text{Overall efficiency, } \eta_o = \frac{\text{water Power}}{\text{shaft Power}} \quad \text{or } \eta_o = \frac{WQH}{P}$$

Slip:

diff b/w Q_{th} & Q_{act} . In most of the cases $Q_{act} < Q_{th}$

$$Q_{act} < Q_{th}$$

negative slip: Some times Q_{act} maybe higher than Q_{th} . In such cases C_d is

$> \text{unity}$ & slip will be negative. It is known as negative slip

Effect of Friction in suction and delivery pipes:

$$h_{fs} = \frac{4f l_s}{2g d_s} \left(\frac{A}{a_s} \omega r \sin \theta \right)^2 \leftarrow \text{for suction pipe}$$

$$h_{fd} = \frac{4f l_d}{2g d_d} \left(\frac{A}{a_d} \omega r \sin \theta \right)^2 \leftarrow \text{for delivery pipe}$$

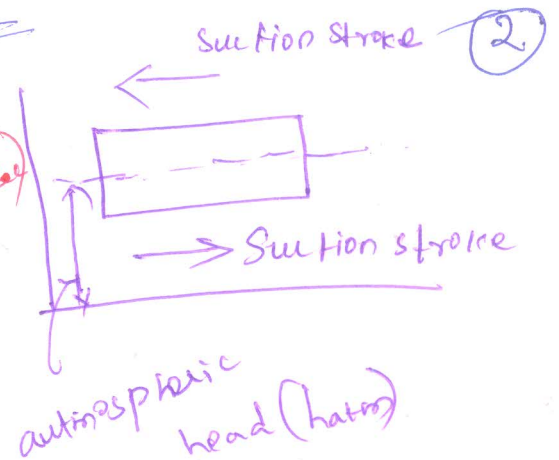
Effect of Both acceleration and friction on indicator diagram in both suction and delivery pipes:

Acceleration head (h_a)

$$h_a = \frac{l}{g} \frac{A}{a} \omega^2 r \cos \theta$$

Friction head (h_f)

$$h_f = \frac{4f l}{2g d} \left[\frac{A}{a} \omega r \sin \theta \right]^2$$



The total pressure head during suction stroke

$$= h_s + h_{as} + h_{fs} \text{ (if given)}$$

$$\text{during delivery} = h_d + h_{ad} + h_{fd}$$

case (i): Suction Stroke

(i) @ the beginning of stroke $\theta = 0^\circ$

$$h_{as} = \frac{l_s}{g} \frac{A}{a_s} \omega^2 r \quad (\cos 0^\circ = 1)$$

$$\text{but } h_{fs} = 0 \leftarrow \sin 0^\circ = 0$$

$$\text{Total pressure head} = h_{atm} - (h_s + h_{as}) \text{ m of water below the atmospheric head}$$

Absolute pressure head

$$h_{fs} = 0 \therefore \text{So}$$

$$\text{pressure head} = h_s + h_{as}$$

* during suction head (h_s) pressure is below the atmospheric pressure
* during delivery head (h_d) pressure is above the atmospheric pressure.

(4)

(ii) @ the middle of the stroke, $\theta = 90^\circ$

$$h_{as} = 0 \quad (\cos 90^\circ = 0)$$

$$h_{fs} = \frac{4fl_s}{2gds} \left[\frac{A}{a_s} \omega r \right]^2 \quad (\because \sin 90^\circ = 1)$$

total pressure head = $H_{atm} - (h_s + h_{fs})$ below the atmospheric head.

(iii) @ the end of the stroke, $\theta = 180^\circ$

$$h_{as} = \frac{-l_s}{g} \frac{A}{a_s} \omega^2 r \quad (\because \cos 180^\circ = -1)$$

$$\text{total head } h_{fs} = 0 \quad (\because \sin 180^\circ = 0)$$

pressure head = $H_{atm} - (h_s + h_{as})$ below the atmospheric head

Case 2 Delivery stroke

✓ Total pressure head during delivery stroke,

$$= h_d + h_{ad} + h_{fd}$$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r \cos \theta$$

(i) @ the beginning of the stroke, $\theta = 0^\circ$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \quad (\because \cos 0^\circ = 1)$$

$$\& h_{fd} = 0 \quad (\because \sin 0^\circ = 0)$$

total pressure head = $H_{atm} + (h_d + h_{ad})$ above the

atmospheric head.

maximum pressure head due to acceleration

$$(h_a)_{\max} = \frac{l_d}{g} \times \frac{A}{a} \omega^2 r$$

Max. value of loss of head due to friction

$$(h_f)_{\max} = \frac{4fl}{2gds} \times \left(\frac{A}{a} \omega r \right)^2$$

(ii) @ the middle of the stroke, $\theta = 90^\circ$

$$h_{ad} = 0 \quad (\cos 90^\circ = 0)$$

$$h_{fd} = \frac{4f l_d}{2g a_d} \left[\frac{A}{a_d} \omega r \right]^2 \quad (\because \sin 90^\circ = 1)$$

Total pressure head = $H_{atm} + (h_d + h_{fd})$ above the atmospheric head

(iii) @ the end of the stroke, $\theta = 180^\circ$

$$h_{ad} = -\frac{l_d}{g} \frac{A}{a_d} \omega^2 r \quad (\because \cos 180^\circ = -1)$$

$$\times h_{fd} = 0 \quad (\because \sin 180^\circ = 0)$$

Total pressure head = $H_{atm} + (h_d - h_{ad})$

work done

$$= \frac{W A L N}{60} (h_s + h_d + h_{fs} + h_{fd}) \quad \text{for single acting pump.}$$

$$= \frac{2 W A L N}{60} (h_s + h_d + h_{fs} + h_{fd}) \quad \text{for double acting pump.}$$

for prob

Effect of acceleration on indicator diagram in Both Suction & delivery Pipes. [1. effect of acc in Suction Pipe]

① Acceleration head at the beginning of the suction stroke.

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \quad (\theta = 0^\circ)$$

pressure head = $h_s + h_{as}$

absolute pressure head = $H_{atm} - (h_s + h_{as})$

below the atm pressure

$$\text{Abs pressure head} = H_{\text{atm}} - h_s + h_{as} \rightarrow 0$$

(6)

(ii) acceleration head at the middle of the suction stroke.

$$\text{acc head } (h_{as}) = 0 \quad (\theta = 90^\circ) \quad h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r \cos \theta \quad h_{as} = 0$$

(iii) acceleration head at the end of the suction stroke.

$\theta = 180^\circ$ at the end of stroke.

$$\text{acceleration head, } h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos \theta$$

$$(\cos 180 = -1)$$

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos (180)$$

$$h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \quad H_{\text{atm}} - (h_s - h_{as})$$

$$\text{absolute pressure head} = H_{\text{atm}} - \left(h_s - \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \right)$$

(2) Effect of acceleration in the delivery pipe:

(i) Absolute pressure at the beginning of the delivery stroke!

$\theta = 0^\circ$ at the beginning of the delivery stroke.

$$\text{The acceleration head, } h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos \theta \quad \theta = 0^\circ$$

$$\cos \theta = 1$$

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$$

A - Area of piston
 a_s - area of pipe

$$\text{Absolute pressure head} = H_{\text{atm}} + \left(h_d + \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \right)$$

(ii) Absolute pressure head at the middle of the delivery stroke.
 $\theta = 90^\circ$ at the middle of the delivery pipe.

$$\text{The acc head, } h_{ad} = 0 \quad (\cos 90 = 0)$$

$$\text{Absolute pressure head} = H_{\text{atm}} + h_d$$

(iii) Absolute pressure head @ the end of delivery stroke!

$$\text{acc head } h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos 180$$

$$\cos 180 = -1$$

$$= -\frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$$

$$\text{Absolute pressure head} = h_{atm} + h_d - \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \quad (7)$$

Effect of variation of velocity on Friction in pipes:

Darcy weisbach equation is

$$h_f = \frac{4fLV^2}{2gd}$$

f - frictional coefficient

L - length of the pipe

d = dia " "

Head loss due to friction in Suction Pipes! V = velocity of liquid in the pipe.

$$h_{fs} = \frac{4fL_s}{2gd} \left[\frac{A}{a_s} \omega r \sin \theta \right]^2 \quad (\because V = \frac{A}{a} \omega r \sin \theta)$$

Head loss due to friction in delivery pipes:

$$h_{fd} = \frac{4fL_d}{2gd} \left(\frac{A}{a_s} \omega r \sin \theta \right)^2$$

(i) at beginning of the stroke $\theta = 0^\circ$

$$\sin 0^\circ = 0$$

$$h_f = 0$$

(ii) @ the middle of the stroke, $\theta = 90^\circ$

$$\sin 90^\circ = 1$$

$$h_f = \frac{4fL}{2gd} \left(\frac{A}{a} \omega r \right)^2 \rightarrow \text{max head loss will occur}$$

(iii) @ the end of the stroke, $\theta = 180^\circ$

in middle stroke.

$$\sin 180^\circ = 0$$

$$h_f = 0$$

~~Effect~~ ^{elevation} of Acc on velocity & pressure head in Both Suction & delivery pipes

(i) at the beginning of the stroke $\theta = 0^\circ$



$$\text{Suction acc head } h_{as} = \frac{L_s}{g} \times \frac{A}{a_s} \omega^2 r$$

$$\cos 0^\circ = 1$$

$$\text{delivery acc head } h_{ad} = \frac{L_d}{g} \times \frac{A}{a_s} \omega^2 r$$

(3 page)

(i) at the middle of the stroke, $\theta = 90^\circ$

$$h_{as} = 0; \quad h_{ad} = 0$$

$$\cos 90 = 0$$

(ii) at the end of the stroke $\theta = 180^\circ$

$$h_{as} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$$

$$\cos 180 = -1$$

$$h_{ad} = -\frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r$$

maxi pressure head due to acceleration.

$$(h_a)_{\max} = \frac{l}{g} \times \frac{A}{a} \omega^2 r$$

(When $\theta = 0^\circ$)

① A single acting

D. A single acting reciprocating pump has a diameter (piston) of 120 mm and stroke length of 300 mm. The centre of the pump is 3.0 m above the water surface in the sump & 20 m below the delivery water level. Both the suction & delivery pipes have the same diameter of 75 mm & 5 m & 10 m long respectively. If the pump is working at 35 rpm determine.

- The pressure heads on the piston at the beginning, middle & end of both suction & delivery strokes.
- The power required to drive the pump
- work done

Take atmospheric pressure as 10.3 m of water
 given data:

Single acting Reciprocating Pump

diameter of the piston, $D = \frac{120 \text{ mm}}{10^3} = 0.12 \text{ m}$

stroke of the piston, $L = \frac{300 \text{ mm}}{10^3} = 0.3 \text{ m}$

Suction head, $h_s = 3 \text{ m}$

delivery " $h_d = 20 \text{ m}$

diameter of pipes $d_s = d_d = \frac{75 \text{ mm}}{10^3} = 0.075 \text{ m}$

Length of Suction pipe, $l_s = 5 \text{ m}$

" " delivery " $l_d = 10 \text{ m}$

speed of pump, $N = 35 \text{ rpm}$

Atmospheric pressure head, $H_{atm} = 10.3 \text{ m}$

To find:

- pressure heads @ beginning & middle & end of both suction & delivery strokes
- Power (P)
- work done

① pressure heads on Suction Pipes:

(a) at the beginning of the suction stroke: $\theta = 0^\circ$

The acceleration head,

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos \theta$$

$$= \frac{5}{9.81} \times \frac{0.011309}{4.4178 \times 10^{-3}} \times (3.6651)^2 \times 0.15 \times \cos(0)$$

$$= 0.5096 \times 2.5578 \times 2.0149$$

$$h_{as} = 2.6284 \text{ m of water}$$

$\leftarrow h_s + h_{as} + h_{fs}$

Pressure head = $h_s + h_{as}$

$$= 3 + 2.6284$$

$$h_s + h_{as} = 5.6284 \text{ m of water}$$

$\therefore$ Absolute pressure head

$$= H_{atm} - (h_s + h_{as})$$

$$= 10.3 - [5.6284]$$

$$= 4.67 \text{ m of water}$$

$$A = \frac{\pi}{4} D^2$$

$$= \frac{\pi}{4} \times (0.12)^2$$

$$A = 0.011309 \text{ m}^2$$

$$a = \frac{\pi}{4} (d_s)^2 = \frac{\pi}{4} \times (0.075)^2$$

$$a = 4.4178 \times 10^{-3} \text{ m}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 35}{60}$$

$$\omega = 3.6651 \text{ rad/s}$$

$$L = 2r = \frac{L}{2} = r$$

$$= \frac{0.3}{2} = 0.15 \text{ m}$$

$$r = 0.15 \text{ m}$$

(b) At the middle of the suction stroke: $(\theta = 90^\circ)$

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos [90^\circ]$$

$$\cos 90^\circ = 0$$

$$h_{as} = 0$$

$$\text{absolute pressure} = H_{atm} - [h_s + h_{as}]$$

$$= 10.3 - [3]$$

$$= 7.3 \text{ m of water}$$

at the end of the Suction stroke! ($\theta = 180^\circ$)

(3)

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos(180^\circ)$$

$$\cos(180) = -1$$

$$= - \frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r$$

$$= - \frac{5}{9.81} \times \frac{0.011309}{4.4178 \times 10^{-3}} \times (3.6651)^2 \times 0.15$$

$$= -0.5096 \times 2.5704 \times 2.0149$$

$$h_{as} = -2.6393 \text{ m of water}$$

$$\text{pressure head} = h_s + h_{as} = 3 - 2.693$$

$$= 0.3606 \text{ m of water}$$

$$\text{Absolute pressure head} = H_{atm} - (h_s + h_{as})$$

$$= 10.3 - (0.3606)$$

$$\text{APH} = 9.9393 \text{ m of water}$$

2. pressure head on the delivery pipe:

a) At the beginning of delivery stroke: ($\theta = 0^\circ$)

$$\text{Acceleration head } h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos \theta$$

$$\cos(0) = 1$$

$$h_{ad} = \frac{10}{9.81} \times \frac{0.011309}{4.4178 \times 10^{-3}} \times (3.6651)^2 \times 0.15 \times \cos(0)$$

$$A = \frac{\pi}{4} D^2 = 0.011309 \text{ m}^2$$

$$= 1.0193 \times 2.5704 \times 2.0149$$

$$a_d = \frac{\pi}{4} d_d^2 = 4.4178 \times 10^{-3} \text{ m}^2$$

$$h_{ad} = 5.279 \text{ m of water}$$

$$\omega = \frac{2\pi N}{60} \text{ r/s}$$

$$\omega = 3.6651 \text{ r/s}$$

$$\text{Pressure head} = h_d + h_{ad} = 20 + 5.275$$

$$= 25.275 \text{ m of water}$$

$$\therefore \text{Absolute pressure head} = H_{atm} + (h_d + h_{ad})$$

$$= 10.3 + 25.275 = 35.575 \text{ m of water}$$

b) At the middle of delivery stroke: ($\theta = 90^\circ$)

(4)

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos \theta$$

$$\cos(90) = 0$$

$$\boxed{h_{ad} = 0}$$

$$-h_d + h_{ad} \rightarrow 0$$

$$\text{pressure head} = h_d = 20 \text{ m}$$

$$\text{Absolute pressure} = H_{atm} + [h_d + h_{ad}]$$

$$= 10.3 + [20 + 0]$$

$$\boxed{= 30.3 \text{ m of water}}$$

c) At the end of delivery stroke: ($\theta = 180^\circ$)

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos(180^\circ)$$

$$\cos 180 = -1$$

$$= -\frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r$$

$$= -\frac{10}{9.81} \times \frac{0.011309}{4.4178 \times 10^{-3}} \times (3.6651)^2 \times 0.15$$

$$\boxed{h_{ad} = -5.26 \text{ m of water}}$$

$$\text{pressure head} = h_d + h_{ad} = 20 - 5.26 = 14.74$$

$$= 14.74 \text{ m of water}$$

$$\text{Absolute pressure head} = H_{atm} + (h_d + h_{ad})$$

$$= 10.3 + (14.74)$$

$$\boxed{= 25.04 \text{ m of water}}$$

② Power required to drive the pump (P): $\frac{\text{work done / sec}}{1000} \text{ in kW}$

$$P = \frac{\rho g A L N}{60 \times 1000} \times (h_s + h_d) \text{ in kW} \quad \bigg| \quad P = W Q$$

$$= \frac{1000 \times 9.81 \times 0.011309 \times 0.3 \times 35}{60 \times 1000} \times [3 + 20]$$

(1008 pg)

⑤

$$P = 0.4465 \text{ kW}$$

$$P = 446.53 \text{ W}$$

③ work done :

$$= W Q \times H_{\text{tot}}$$

$$= W Q (h_s + h_f)$$

$$= e g \times Q (h_s + h_f)$$

$$= 1000 \times 9.81 \times 1.9791 \times 10^{-3}$$

$$\times [3 + 20]$$

$$\text{work done} = 446.53 \text{ J}$$

$$H_{\text{tot}} = h_s + h_f$$

$$Q = \frac{A L V}{60}$$

$$= \frac{0.011307 \times 0.3 \times 35}{60}$$

$$Q = 1.97907 \times 10^{-3} \text{ m}^3/\text{s}$$

Hydraulic Machines - Turbines

Hydraulic machines:

Those machines which convert either hydraulic energy into mechanical energy (converted into electrical energy) (or) mechanical energy into hydraulic energy.

The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called turbines.

convert Mechanical energy into hydraulic energy are called Pumps.

Turbines:

hydraulic machines which convert hydraulic energy into mechanical energy. This mech energy is used in running an electric generator which is directly coupled to the shaft of the turbine.

Thus the mechanical energy is converted into electrical energy. The electric power which is obtained from the hydraulic energy (energy of water) is known as Hydro-electric power.

General layout of a hydro-electric power plant

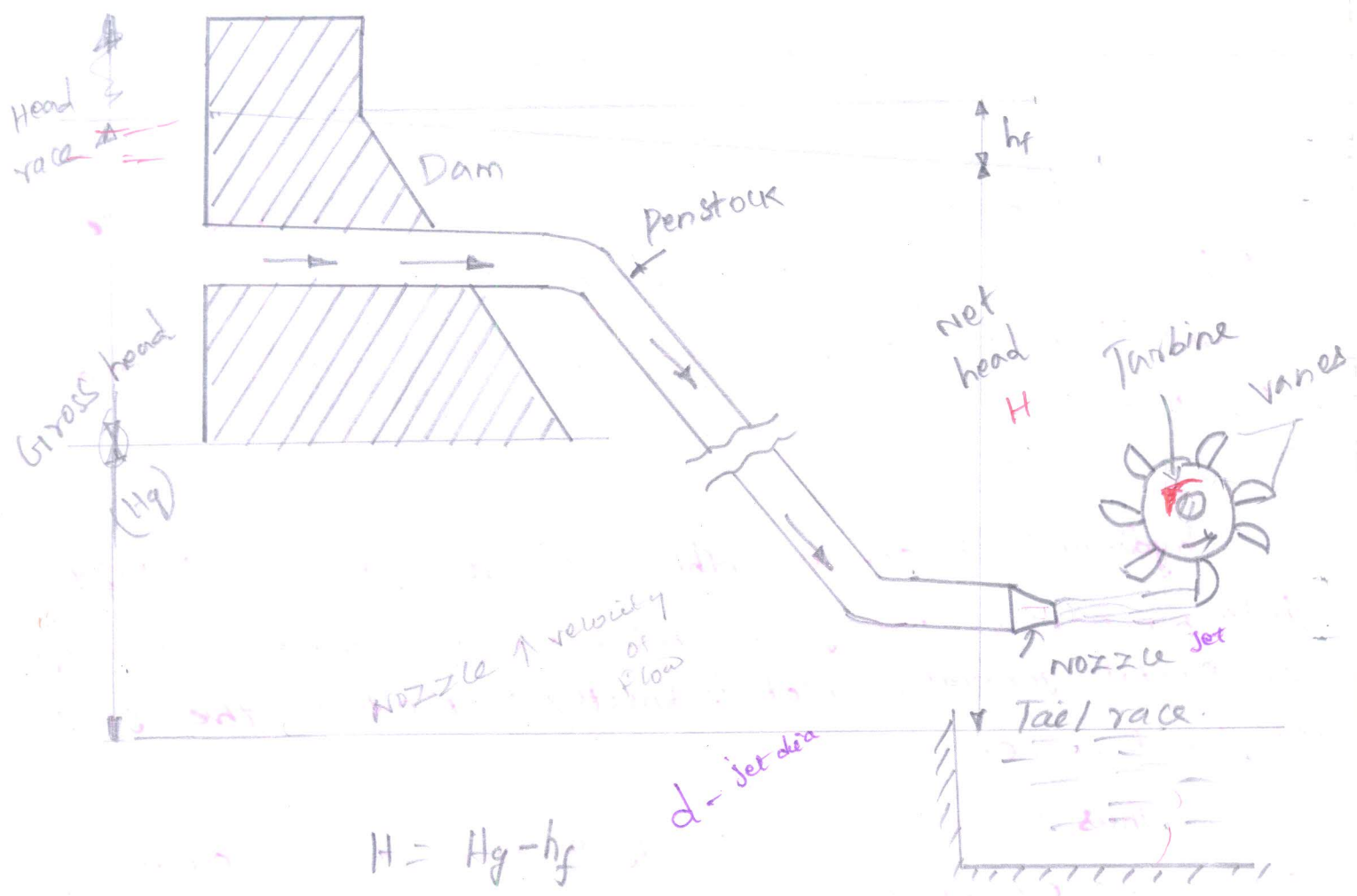
- A dam constructed across a river to store water.
- pipes of large diameters called Penstocks which carry water under pressure from the storage reservoir to the turbines.

The pipes made up of steel (or) ~~rein~~ reinforced concrete

- Turbines having different types of vanes fitted to the wheels

→ Tail race - is a channel which carries water away from the turbines after the water has worked on the turbines.

the surface of water in the tail race is also known as tail race.



$$H_g = \text{Gross head}, h_f = \frac{4flv^2}{2gd}$$

$v \rightarrow$ velocity of flow in penstock

$L \rightarrow$ length of penstock

$D \rightarrow$ diameter " "

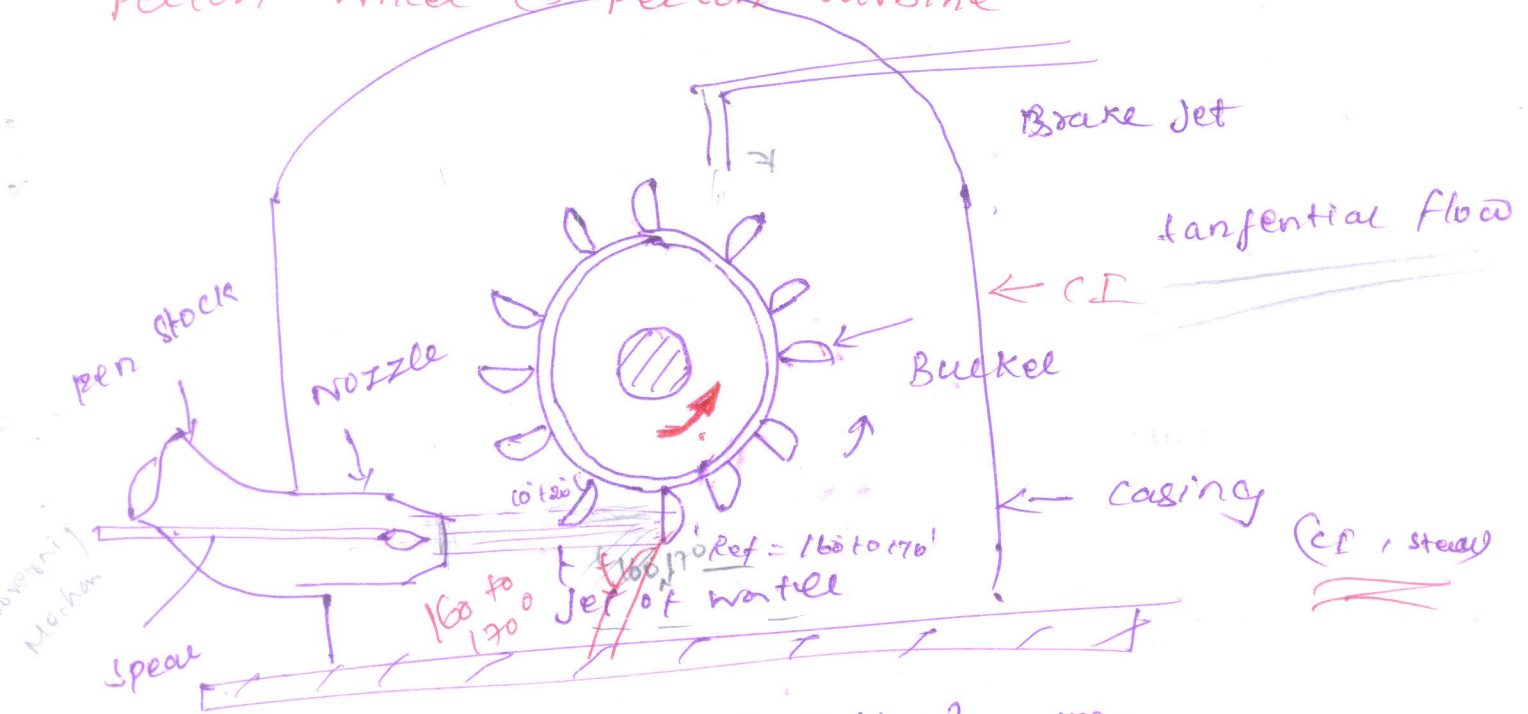
Gross head:

The difference between the head race level and tail race level when no water is flowing is known as gross head. denoted by, (H_g)

$$h_f + H = H_g$$
$$H = H_g - h_f$$



Pelton wheel (or) pelton turbine

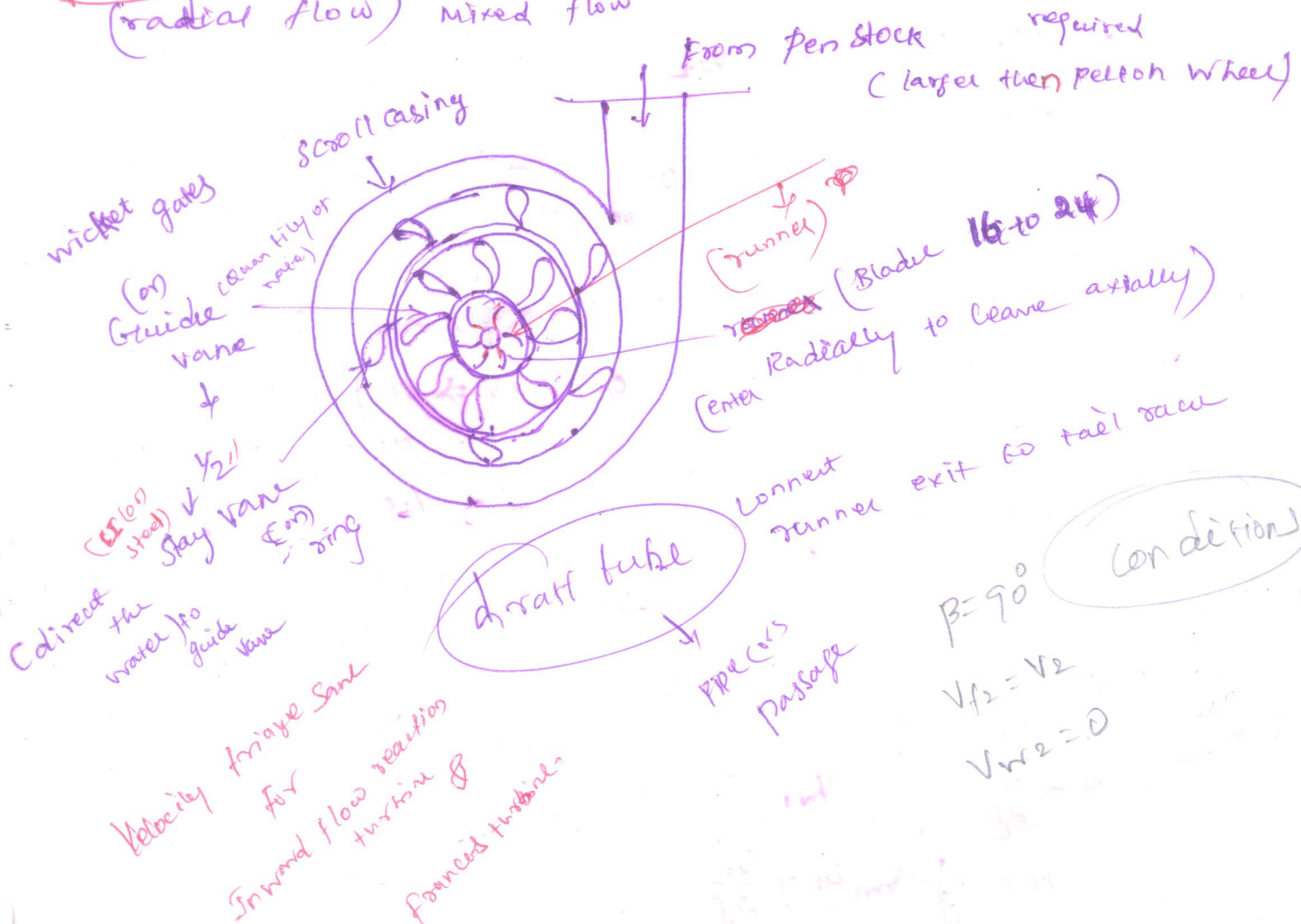


American engineer (Lester A Pelton) 1880

Radial Reaction turbines

Francis Turbine: James B. Francis American engg

Inward flow reaction turbine & Outward
(radial flow) mixed flow



(3) Relative velocity at inlet (V_{r1})

$$V_{r1} = \sqrt{(V_{f1})^2 + (V_{w1} - u_1)^2} = \sqrt{(1.8)^2 + (10.207 - 9.424)^2}$$

$$V_{r1} = 1.963 \text{ m/s}$$

(4) The runner blade angles means triangle θ & ϕ

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1} - u_1} = \frac{1.8}{(10.207 - 9.424)}$$

$$\tan \theta = 2.298 \Rightarrow \theta = \tan^{-1}(2.298)$$

$$\theta = 66^\circ 29'$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.9 \times 200}{60} = 9.424 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 200}{60} = 4.712 \text{ m/s}$$

(5) From outlet velocity triangle, we have

$$\tan \phi = \frac{\text{opp}}{\text{adj}} = \frac{V_{f2}}{u_2} = \frac{1.8}{4.712} = \tan(20.9^\circ) \Rightarrow \phi = 20.9^\circ$$

$$\tan \phi = 0.3820 \Rightarrow \phi = \tan^{-1}(0.3820)$$

$$\phi = 20.90^\circ$$

(5) width of runner at outlet (B_2)

$$Q = A \times V$$

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2} \quad V_{f1} = V_{f2}$$

$$D_1 B_1 = D_2 B_2 \Rightarrow \frac{D_1 B_1}{B_2} = D_2 \Rightarrow \frac{0.9 \times 0.20}{0.45} = 0.40 \text{ m}$$

$$B_2 = 400 \text{ mm}$$

(6) mass of water flowing through the runner / sec (M)

$$\text{Mass} = \rho \times Q \Rightarrow Q = A \times V \Rightarrow \pi D_1 B_1 \times V_{f1}$$

$$M = 1000 \times 1.017 \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}^3}{\text{s}}$$

$$\text{Mass} = 1017.8 \text{ kg/s}$$

$$Q = A \times V \Rightarrow \pi D_1 B_1 \times V_{f1}$$

$$= \pi \times 0.9 \times 0.20 \times 1.8$$

$$Q = 1.017 \text{ m}^3/\text{s}$$

(7) Head at the inlet of turbine [H]

$$H - \frac{V_2^2}{2g} = \frac{1}{g} (V_{w1} u_1 \pm V_{w2} u_2)$$

(Radial flow)
 $V_{w2} = 0$

$$H - \frac{V_2^2}{2g} = \frac{1}{g} (V_{w1} u_1)$$

$$H = \frac{1}{g} V_{w1} u_1 + \frac{V_2^2}{2g}$$



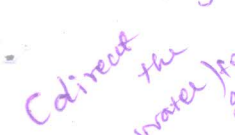
American engineers

Radical Reaction, Carbonyles

Francis Turbick James B. Francis American essay

inward flow reaction turbine & outward

(radial flow) mixed flow



$$\beta = 90^\circ$$

$$V_{f2} = \sqrt{2}$$

$$V_{w2} = 0$$

Con dition

Formulas for Radial reaction turbine

(1)

(or)

Francis turbine (or)

Inward flow reaction turbine

Workdone per second on the runner by water:

$$= \rho Q [V_{w1} u_1 \pm V_{w2} u_2] \quad \therefore Q = aV$$

Workdone per second per unit weight of water per second.

$$= \frac{1}{g} [V_{w1} u_1 \pm V_{w2} u_2]$$

$$= \frac{1}{g} [V_{w1} u_1] \rightarrow \rightarrow$$

$$\beta = 90^\circ \quad V_{w2} = 0$$

PUMP
of 90
V_{w1} = 0

V_{w1} & V_{w2} = Velocity of whirl at inlet & outlet

u₁ & u₂ = Tangential velocity @ Inlet & outlet

$$u_1 = \frac{\pi D_1 N}{60}$$

$$u_2 = \frac{\pi D_2 N}{60}$$

u₁ & u₂ = Tangential velocity @ Inlet & outlet

if radial flow V_{w2} = 0

Pell on
ww
(See next page)

Hydraulic efficiency!

Turbine power
Water power

$$\eta_h = \frac{R.P}{W.P} = \frac{(V_{w1} u_1 \pm V_{w2} u_2)}{gH} = \frac{V_{w1} u_1}{gH}$$

Mechanical efficiency (η_m)

$$\eta_m = \frac{P}{\frac{WQ}{g} (V_{w1} u_1 - V_{w2} u_2)}$$

Overall efficiency (η_o)

$$\eta_o = \frac{\text{Shaft power}}{\text{water power}} = \frac{P}{WQH}$$

$$\text{Power (P)} = \frac{\text{work done/sec on runner}}{1000}$$

$$= \frac{\rho Q [V_{w1} u_1]}{1000} \text{ in kW}$$

Discharge of the turbine

$$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$$

speed Ratio! $K_u = \frac{u_1}{\sqrt{2gH}}$ from 0.6 to 0.9

Flow Ratio!

$$K_f = \frac{V_{f1}}{\sqrt{2gH}}$$

0.15 to 0.3

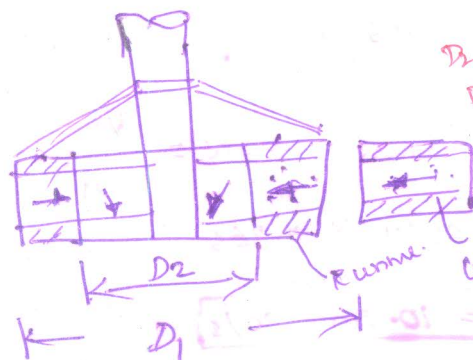
H = head on turbine

$$H = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} \text{ (or) } \frac{1}{g} V_{w1} u_1 + \frac{V_1^2}{2g} = H$$

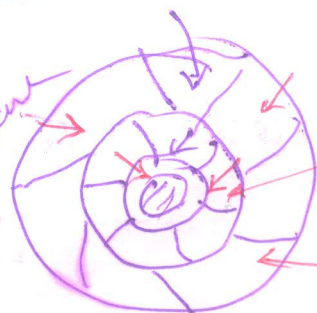
$$\text{(or) } \eta_o = \eta_m \times \eta_h$$

if no losses of energy when water flows through the vanes

Inward radial flow turbine. $V_{w2} = V_2$
 $V_{w2} = 0$



D₂ = Internal dia
D₁ = external dia



(moving vanes)
Runners
guide vane

α - Guide Blade angle
 ϕ & θ = runner blade angle

mass of water flowing through the runner / sec

$$\text{mass} = \rho \times Q \text{ in kg/s}$$

(h_n) Hydraulic h

$$\frac{V_{w1} u_1}{gH} = h_n = \frac{\text{Head at inlet} - \text{hydraulic losses}}{\text{Head at inlet}}$$

Inward Flow

Ex in prob no: 18.15 From Bansal. R. R

Given data:

External dia. $D_1 = 0.9 \text{ m}$

Internal dia $D_2 = 0.45 \text{ m}$

speed $N = 200 \text{ rpm}$

width at inlet $B_1 = 200 \text{ mm} / 1000 = 0.2 \text{ m}$

Velocity of flow $V_{f1} = V_{f2} = 1.8 \text{ m/s}$

Guide blade angle $\alpha = 10^\circ$

discharge at outlet = Radial $\beta = 90^\circ$ & $V_{w2} = 0$

$$V_{f2} = V_2$$

To Find:

(1) The absolute velocity of water at inlet of runner.

(2) The velocity of whirl at inlet (3) The relative velocity at inlet (4) The runner blade angles

(5) width of the runner at outlet

(6) mass of water flowing through the runner per second.

(7) Head at the inlet of the runner. (8) Power developed & hydraulic h of turbine.

Solution:

(1) Absolute velocity of water at inlet of the runner, (V_1)

$$\sin \alpha = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin \alpha = \frac{V_{f1}}{V_1} \Rightarrow \sin(10) = \frac{1.8}{V_1}$$

$$V_1 = \frac{1.8}{\sin(10)} \Rightarrow \boxed{V_1 = 10.365 \text{ m/s}}$$

(2) Velocity of whirl at inlet (V_{w1}): $\cos \alpha = \frac{V_{w1}}{V_1} \Rightarrow \cos \alpha \times V_1 = V_{w1}$

$$\cos \alpha \times V_1 = V_{w1} \Rightarrow \cos(10) \times 10.365 = V_{w1}$$

$$\boxed{V_{w1} = 10.207 \text{ m/s}}$$

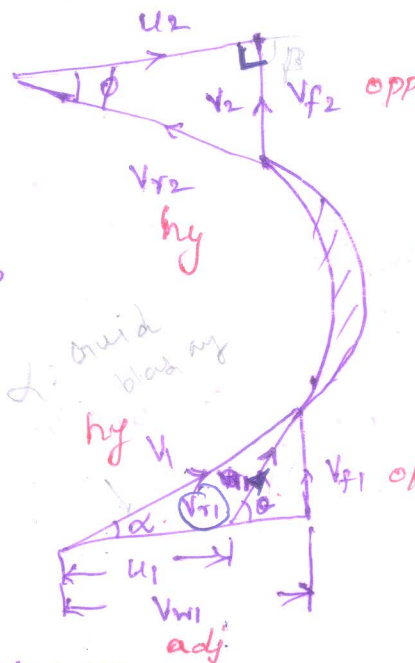
Degree of Reaction (R)

$$R = 1 - \frac{\cot \alpha}{2(\cot \alpha - \cot \theta)}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

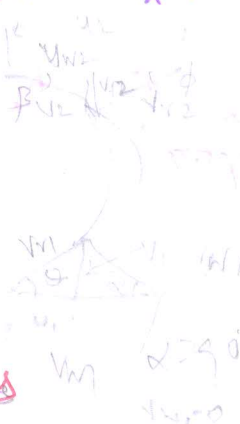
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$



runner angles

$\alpha \leftarrow$ Guide blade angle



$$= \frac{1}{9.81} \times 10.207 \times 9.424 + \frac{(1.8)^2}{2 \times 9.81}$$

$$(V_2 = V_{f2})$$

$$H = 9.97 \text{ m}$$

(8) Power developed: $(P) = \frac{\text{work done / se on runner}}{1000}$

$$= \frac{\rho Q [V_{w1} u_1]}{1000}$$

$$= \frac{1000 \times 1.0178 [10.207 \times 9.424]}{1000}$$

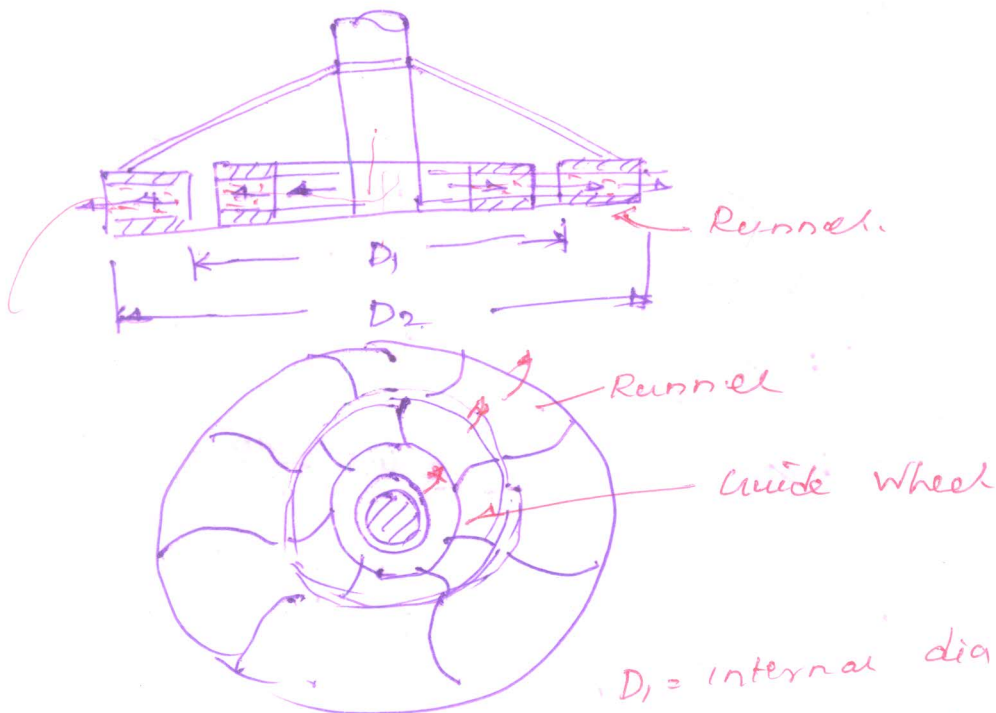
$$P = 97.9 \text{ kW}$$

(9) Hydraulic efficiency $(\eta_h) = \frac{V_{w1} u_1}{gH} = \frac{10.207 \times 9.424}{9.81 \times 9.97}$

for centrifugal pump
 $\eta_h = \frac{gH}{V_{w1} u_1}$

$$\eta_h = 98.34 \%$$

outward Radial Flow Reaction Turbine:



D_1 = internal dia

D_2 = external dia

Given data: outward flow reaction turbine

Internal dia $D_1 = 0.6 \text{ m}$

External dia $D_2 = 1.2 \text{ m}$

Guide blade angle $\alpha = 15^\circ$

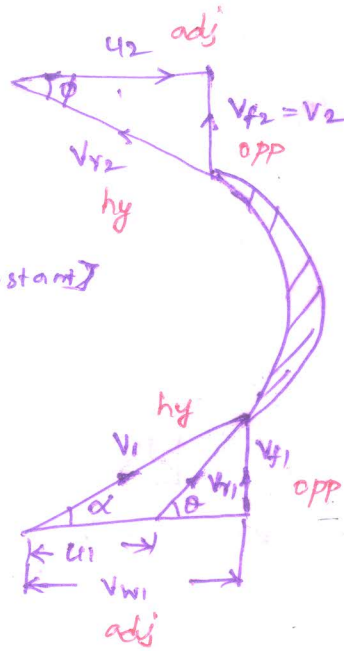
Velocity of flow $V_{f1} = V_{f2} = 4 \text{ m/s}$ [constant]

Speed $N = 200 \text{ rpm}$

Head $H = 10 \text{ m}$

discharge at outlet $\equiv$ Radial $\beta = 90^\circ$

$$V_{w2} = 0, V_{f2} = V_2$$



To Find:

- 1) The runner vane angles @ inlet & outlet (θ) (ϕ)
- 2) work done by the water on the runner / sec / unit weight of water striking / sec
- 3) Hydraulic η $[h_h]$
- 4) The degree of reaction.

Solution:

1) runner vane angles @ Inlet & outlet

Inlet $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1} - u_1}$

$$\tan \theta = \frac{4}{6.283 - 14.928} \quad \tan \theta = \frac{4}{(14.928 - 6.283)}$$

$$\tan \theta = 0.4627$$

$$\theta = \tan^{-1}(0.4627)$$

$$\theta = 24.829^\circ$$

outlet $\tan \phi = \frac{\text{opp}}{\text{adj}} = \frac{V_{f2}}{u_2} = \frac{4}{12.566}$

$$\tan \phi = 0.3183$$

$$\phi = \tan^{-1}(0.3183)$$

$$\phi = 17.65^\circ$$

$$\sin \theta = \frac{\text{opp}}{hy}$$

$$\cos \theta = \frac{\text{adj}}{hy}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.6 \times 200}{60}$$

$$u_1 = 6.283 \text{ m/s}$$



$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1} - u_1}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1}$$

$$V_{w1} = \frac{V_{f1}}{\tan \theta}$$

$$V_{w1} = \frac{4}{\tan(15^\circ)}$$

$$V_{w1} = 14.928 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60}$$

$$u_2 = 12.566 \text{ m/s}$$

② work done by water/sec/unit weight

$$= \frac{1}{g} V_{w1} u_1$$

Radial $V_{w2} = 0$

$$= \frac{1}{9.81} \times 14.928 \times 6.283$$

$$\boxed{WD = 9.561 \text{ Nm/N}}$$

weight = $1 \text{ kg} \Rightarrow 1 \text{ kg} \times 9.81 = \text{N}$

③ Hydraulic efficiency:

$$h_h = \frac{V_{w1} u_1}{gH} = \frac{14.928 \times 6.283}{9.81 \times 10} = 0.9361 \times 100$$

$$\boxed{h_h = 93.61 \%}$$

④ Degree of Reaction

$$R = 1 - \frac{\cot \alpha}{2(\cot \alpha - \cot \theta)}$$

Ans = 0.3

④ Degree of Reaction

② A Francis turbine with an overall efficiency of 70% is required to produce 147.15 kW. It is working under a head of 8m, the peripheral velocity = $0.30 \sqrt{2gH}$ and the radial velocity of the flow at inlet is $0.96 \sqrt{2gH}$. The wheel runs at 200 rpm & the hydraulic losses in the turbine are 20% of the available energy. Assume radial discharge, determine, (i) guide blade angle, (ii) wheel vane angle at inlet, (iii) diameter of wheel at inlet & (iv) width of wheel at inlet. Draw the suitable velocity triangle.

Given data:

Overall efficiency $\eta_o = 70\%$
 $= 0.75$

Power produced (S.P) = 147.15 kW

Head $H = 8 \text{ m}$

Peripheral (or) Tangential velocity $V_{u1} = 0.30 \sqrt{2gH}$

velocity of flow @ inlet $V_{f1} = 0.96 \sqrt{2gH}$

speed $N = 200 \text{ rpm}$

Hydraulic losses = 20% of available energy (H)

Assume radial discharge = $V_{w2} = 0$, $V_{f2} = V_2$

To Find

- 1) The guide blade angle (2)
- 2) The wheel vane angle at inlet (A)
- 3) diameter of the wheel at inlet (D)
- 4) width of the wheel at inlet (B)
- 5) Draw velocity triangle

$100 - 20 = 80\%$
 $\eta_h = 0.8$

① Guide blade angle (α)

$$\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1}}$$

$$h_h = \frac{\text{Total head at inlet} - \text{Hydraulic loss}}{\text{Head at inlet}} \quad (1)$$

$$= \frac{H - 0.2H}{H}$$

$$= \frac{H - 0.2H}{H}$$

$$= 1 - \frac{0.2H}{H}$$

$$\boxed{h_h = 0.8} \Rightarrow h_h = \frac{V_{w1} u_1}{gH} \Rightarrow 0.8 = \frac{V_{w1} \times u_1}{9.81 \times 8}$$

$$0.8 = \frac{V_{w1} \times 3.7585}{9.81 \times 8} \Rightarrow 0.8 \times 9.81 \times 8 = V_{w1} \times 3.7585$$

$$62.784 = V_{w1} \times 3.7585$$

$$V_{w1} = \frac{62.784}{3.7585} \Rightarrow \boxed{V_{w1} = 16.7045 \text{ m/s}}$$

$$\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1}} = \frac{12.027}{16.7045}$$

$$\tan \alpha = 0.7199 \Rightarrow \alpha = \tan^{-1}(0.7199)$$

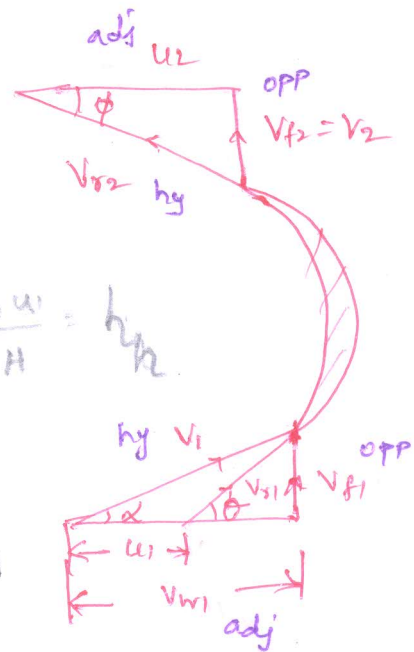
$$\boxed{\alpha = 35.75^\circ}$$

② Wheel vane angle at inlet, θ

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1} - u_1} \Rightarrow \tan \theta = \frac{12.027}{16.7045 - 3.7585}$$

$$\tan \theta = 0.9290 \Rightarrow \theta = \tan^{-1}(0.9290)$$

$$\boxed{\theta = 42.89^\circ}$$



$$u_1 = 0.30 \sqrt{2gH}$$

$$= 0.30 \sqrt{2 \times 9.81 \times 8}$$

$$\boxed{u_1 = 3.7585 \text{ m/s}}$$

$$V_{f1} = 0.96 \sqrt{2gH}$$

$$= 0.96 \sqrt{2 \times 9.81 \times 8}$$

$$\boxed{V_{f1} = 12.027 \text{ m/s}}$$

(7)

(3) diameter of wheel at inlet (D_1):

$$u_1 = \frac{\pi D_1 N}{60} \Rightarrow 3.7585 = \frac{\pi D_1 \times 200}{60}$$

$$3.7585 \times 60 = \pi D_1 \times 200$$

$$\frac{3.7585 \times 60}{\pi \times 200} = D_1 \Rightarrow \boxed{D_1 = 0.3589 \text{ m}}$$

(4) width of wheel at inlet (B_1)

if outlet means

$$Q = A \times V = \pi D_1 B_1 \times V_{f1}$$

$$Q = \pi D_2 B_2 \times V_{f2}$$

$$\eta_o = \frac{S.P}{W.P} = \frac{S.P \text{ kW}}{\frac{e g Q H}{1000} \text{ kW}}$$

$$0.70 = \frac{147.15 \times 1000}{e g Q H}$$

$$0.70 = \frac{147.15 \times 1000}{1000 \times 9.81 \times Q \times 8}$$

$$Q = \frac{147.15 \times 10^3}{1000 \times 9.81 \times 8 \times 0.70}$$

$$\boxed{Q = 2.6785 \text{ m}^3/\text{s}}$$

$$Q = \pi D_1 B_1 \times V_{f1}$$

$$\frac{Q}{\pi D_1 V_{f1}} = B_1 \Rightarrow \frac{2.6785}{\pi \times 0.3589 \times 12.027}$$

$$\boxed{B_1 = 0.1975 \text{ m}}$$

18.24 HW

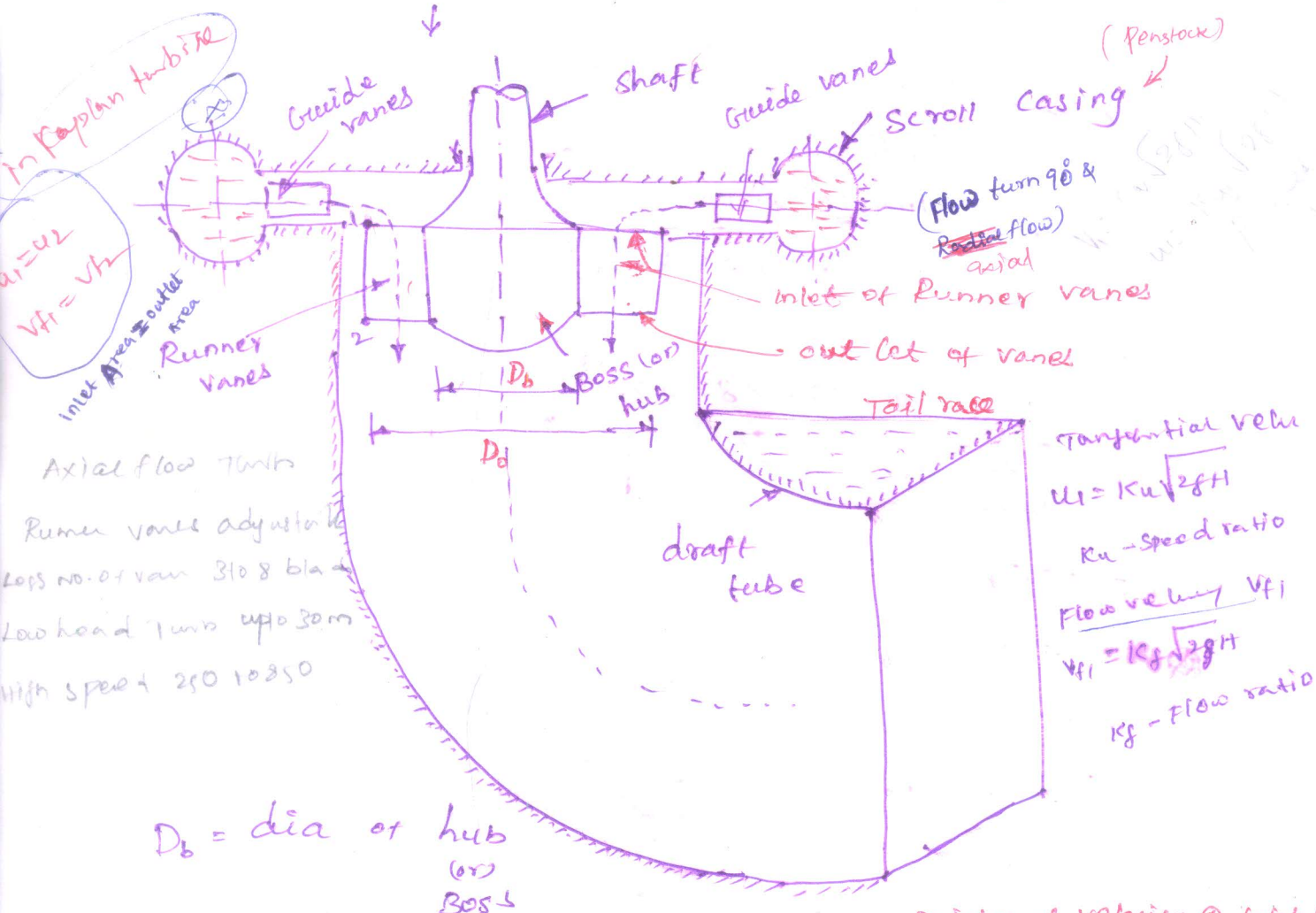
Axial flow Reaction Turbine (or ~~Kaplan~~ Turbine).

→ If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine

Reaction turbine:

during flow of water through runner a part of pressure energy is converted into kinetic energy the turbine is known as reaction turbine.

- Propeller Turbine - vanes fixed to hub & not adjustable
- Kaplan Turbine - vanes on the hub are adjustable



Axial flow Turbines
 Runner vanes adjustable
 Less no. of vanes 31 to 8 blades
 Low head runs up to 30m
 High speed 250 to 10850

Tangential velocity
 $u_1 = K_u \sqrt{2gH}$
 K_u - Speed ratio
 Flow velocity V_{f1}
 $V_{f1} = K_f \sqrt{2gH}$
 K_f - Flow ratio

D_b = dia of hub
 (or)
 Boss

V_{f1} = velocity of flow @ inlet

D_0 = outer dia of runner

$$Q = A \times V \Rightarrow \frac{\pi}{4} (D_0^2 - D_b^2) \times V_{f1}$$

Water Power = $\frac{\rho g Q H}{1000}$

Peripheral velocity @ inlet & outlet are equal
 outer dia

$$u_1 = u_2 = \frac{\pi D_0 N}{60}$$

Velocity of flow at inlet & outlet are equal

$$V_{f1} = V_{f2}$$

$$A_{inlet} = A_{outlet}$$

From Bansal R.K. Prob. no. 18.27

Given data Kaplan turbine:

Head $H = 20 \text{ m}$

Shaft power $S.P = 11,772 \text{ kW}$

outer dia of runner $D_o = 3.5 \text{ m}$

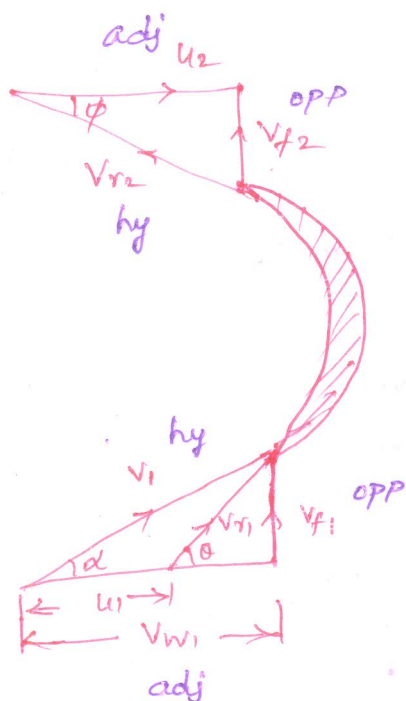
Hub dia $D_b = 1.75 \text{ m}$

Guide blade angle $\alpha = 35^\circ$

Hydraulic efficiency, $\eta_h = 88\%$

Overall $\eta_o = 84\%$

Velocity of whirl at outlet $= 0$
(V_{w2})



To find:

(i) Runner vane angle @ inlet & outlet

(ii) Speed of the turbine.

$$\sin \theta = \frac{\text{opp}}{\text{hy}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hy}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

Solution

① Runner vane angle @ inlet & outlet (θ & ϕ)

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1} - u_1}$$

for find V_{f1}

$$Q = A \times V \Rightarrow \frac{\pi}{4} (D_o^2 - D_b^2) \times V_{f1}$$

$$\eta_o = \frac{S.P}{W.P} = \frac{11772 \text{ kW}}{\left(\frac{\text{eg } Q \times H}{1000} \right) \text{ in kW}}$$

$$\frac{\text{eg } Q \times H}{1000} = \text{kW} \Rightarrow \times 1000$$

$$\text{eg } Q \times H = W$$

$$0.84 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20}$$

$$Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 20 \times 0.84}$$

$$Q = 71.428 \text{ m}^3/\text{s}$$

$$Q = \frac{\pi}{4} (D_o^2 - D_b^2) \times V_{f1}$$

$$71.428 = \frac{\pi}{4} ((3.5)^2 - (1.75)^2) \times V_{f1}$$

$$71.428 = 7.216 \times V_{f1}$$

$$V_{f1} = \frac{71.428}{7.216} \Rightarrow \boxed{V_{f1} = 9.9 \text{ m/s}}$$

for find V_{w1}

$$\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1}}$$

$$\tan 35 = \frac{9.9}{V_{w1}} \Rightarrow V_{w1} = \frac{9.9}{\tan 35}$$

$$\boxed{V_{w1} = 14.14 \text{ m/s}} \quad \checkmark$$

for find u_1

$$h_h = \frac{V_{w1} \times u_1}{gH} \Rightarrow 0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$\frac{0.88 \times 9.81 \times 20}{14.14} = u_1 \Rightarrow \boxed{u_1 = 12.21 \text{ m/s}}$$

for find θ $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{V_{f1}}{V_{w1} - u_1}$

$$\tan \theta = \frac{9.9}{14.14 - 12.21} \Rightarrow \tan \theta = 5.13$$

$$\theta = \tan^{-1}(5.13) \Rightarrow \boxed{\theta = 78.97^\circ} \quad \checkmark$$

for find ϕ

$$\tan \phi = \frac{\text{opp}}{\text{adj}} = \frac{V_{f2}}{u_2}$$

for Kaplan turbine $u_1 = u_2$, $V_{f1} = V_{f2}$

$$\tan \phi = \frac{9.9}{12.21}$$

$$\tan \phi = 0.811$$

$$\phi = \tan^{-1}(0.811)$$

$$\boxed{\phi = 39.035^\circ}$$

② Speed of turbine:

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

peripheral velocity

$$12.21 = \frac{\pi \times 3.50 \times N}{60}$$

$$\frac{12.21 \times 60}{\pi \times 3.50} = N$$

$$\boxed{N = 66.63 \text{ rpm}}$$

Draft Tubes

- conical
- simple elbow tubes
- Moody spreading tubes
- Elbow draft tube in circular inlet & rectangular outlet

Formulas Pelton wheel Turbine

①

Hydraulic Efficiency (η_h)

$$\eta_h = \frac{\text{Power delivery to runner}}{\text{Power supplied at inlet}} = \frac{R.P}{W.P}$$

R.P $\rightarrow$ Runner power

W.P $\rightarrow$ water power

$$R.P = \frac{W [V_{w1} \pm V_{w2}] \times u}{g \times 1000} \text{ kW for pelton Turbine}$$

$$= \frac{W}{g} \times \frac{[V_{w1} u_1 \pm V_{w2} u_2]}{1000} \text{ kW for a radial flow turbine}$$

$$W.P = \frac{W \times H}{1000} \text{ kW} \quad W = WQ$$

$$W = \rho g \times Q \quad \checkmark$$

$$\omega = \rho g$$

H $\leftarrow$ net head on the turbine

u_1 & $u_2 \leftarrow$ Tangential velocity of vane at inlet for radial vane @ I/L & O/L

V_{w1} & V_{w2} - velocity of whirl @ I/L & O/L

$$W.P = \frac{\rho g Q \times H}{1000} \quad \checkmark$$

$$= \frac{1000 \times \cancel{\rho} g \times Q \times H}{1000} \quad \therefore \rho = 1000 \text{ kg/m}^3$$

$$W.P = g Q H \text{ kW}$$

$$\frac{\rho g Q H}{1000} = W.P \rightarrow \text{ICW}$$

Mechanical Efficiency (η_m):

$$\eta_m = \frac{S.P \leftarrow (P)}{R.P} \quad \text{Runner power}$$

$$\text{Speed ratio } (K_u) = \frac{u}{V_1} \quad \begin{matrix} \leftarrow \text{velocity of wheel} \\ \leftarrow \text{velocity} \end{matrix}$$

Volumetric efficiency (η_v)

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine.}}$$

$$\eta_o = \frac{\text{water power}}{\text{shaft power}} \quad (\text{its for pumps}) \quad (2)$$

Overall Efficiency (η_o):

$$\frac{WP}{IP} = \eta_o = \frac{\text{Shaft power (its for turbine)}}{\text{water power}}$$

(S.P)

P = Shaft power

speed ratio (K_u)

$$\eta_o = \frac{P}{\left(\frac{\rho g Q H}{1000} \right)}$$

$$u = \phi \sqrt{2gH}$$

velocity of wheel

$$Q(1-K_u) = \frac{u}{V}$$

D^* = Dia of penstock

D = dia of the wheel

d = dia of the jet

$$h_f = \frac{4fLv^2}{2gD^5}$$

K_{v1}
(or)

$$V_1 = C_v \sqrt{2gH}$$

V_1 = velocity of jet at inlet

C_v = co-efficiency of velocity 0.98 to 0.99

H = Net head on turbine

$$u = u_1 = u_2 = \frac{\pi DN}{60}$$

Force Exerted by the jet of water (F_x)

$$F_x = \rho a V_1 [V_{w1} + V_{w2}]$$

$$Q = a V_1$$

work done by the jet on the runner Per Second

$$\text{Power develop} = F_x \times u \Rightarrow \rho a V_1 [V_{w1} + V_{w2}] \times u \text{ in Nm/s}$$

work done/s per unit weight of water striking/s

$$= \frac{1}{g} [V_{w1} + V_{w2}] \times u \text{ Nm/N}$$

$$\text{weight} = \rho g \times 9.81 = \rho V$$

$$(K.E) \text{ kinetic energy of jet/s} = \frac{1}{2} (\rho a V_1) \times V_1^2$$

Hydraulic efficiency [η_h]

$$\eta_h = \frac{\text{work done Per Second}}{\text{K.E of jet / sec}}$$

$$h_h = \frac{2 [V_{w1} + V_{w2}] \times u}{V_1^2} \quad (\text{or})$$

$$\frac{2(V_1 - u) [1 + \cos \phi] u}{V_1^2} \quad (3)$$

$$\text{Max. } h_h = \frac{2 \times u (1 + \cos \phi) \times u}{V_1^2}$$

$$u = \frac{V_1}{2}$$

$$\text{min } h_h = \frac{(1 + \cos \phi)}{2}$$

$$\text{Power} = \frac{F_x \times u}{1000} \text{ in kW}$$

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}] \times u}{1000} \text{ in kW}$$

$$\text{discharge } Q = a \times V_1 = \frac{\pi}{4} d_1^2 \times V_1$$

$$h_o = \frac{sp}{w.p}$$

$$w.p = \frac{sp \times 11}{1000}$$

$$\rightarrow \text{No. of Jet} = \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q}$$

$$\text{Discharge of jet (q)} = \text{Area of jet} \times \text{velocity of jet } V_1 = C_v \sqrt{2gH}$$

$$q = A \times V \text{ in m}^3/\text{s}$$

$$\text{Net head (H)} = H_g - H_f \leftarrow \text{Frictional head}$$

↓ Gross head

$$N = \frac{60 \times f}{p}$$

$$f = \text{Frequency in Hertz / Sec}$$

$$p = \text{pairs of poles}$$

$$N = \text{Speed}$$

Design Aspects of Pelton wheel (or) Turbine.

(4)

tangential flow

1. Velocity of jet: ✓

$$V_1 = C_v \sqrt{2gH}$$

C_v = Co-efficient of velocity = 0.98 to 0.99

H = Net head of turbine = $H_g - h_f$

2. Velocity of wheel (u) ✓

(or) Pitch dia velocity

$$u = K_u \sqrt{2gH}$$

$$K_u = \text{or } \frac{u}{\sqrt{2gH}}$$

Speed ratio = 0.43 to 0.48

3. Mean dia of wheel [D]

(or) Pitch dia

$$u = \frac{\pi D N}{60}$$

4. Jet ratio (m) ✓

$$m = \frac{D}{d}$$

m varies b/w 11 to 15 normally

m = 12

d - Jet dia

D - wheel dia

5. Number of jets: ✓

1 nozzle on one jet

for one pelton wheel → 6 nozzle theoretically

$$\text{No. of jet} = \frac{Q}{q} = \frac{\text{Total discharge}}{\text{discharge of one jet}} \rightarrow q = A \times V$$

area of jet
velocity of jet

6. Bucket dimensions: ✓

width of the buckets (B) = $5 \times d$ (or) $4d$

depth " " (T) = $1.2d$

Radial length (L) = $0.8d$ to $1.2d$
→ $2d$ to $3d$

7. Number of buckets (Z) ✓

principles

1) no. of buckets should be few to reduce the friction loss

2) NO water escapes without striking the bucket.

$$Z = \frac{D}{2d} + 15$$

$$= 0.5m + 15$$

$$m = \frac{D}{d}$$

Jet ratio

Design of Pelton turbine

- 1) design a Pelton wheel for a head of 400m when running at 750 rpm. The pelton wheel develops 12,110 kW shaft power. The ratio of jet diameter to the wheel diameter is $\frac{1}{6}$. The overall efficiency $\eta_o = 0.86$, coefficient of velocity $C_v = 0.985$ and speed ratio $\phi = 0.45$

Given data:

head $H = 400\text{ m}$

speed $N = 750\text{ rpm}$

S.P. $= 12,110\text{ kW} = 12,110 \times 10^3\text{ W}$

To Find

- ① Design a Pelton wheel

Jet ratio $\frac{d}{D} = \frac{1}{6}$

$\eta_o = 0.86$

$C_v = 0.985$

Speed ratio (ϕ) $= 0.45$

Solution:

1. velocity of jet:

$$V_j = C_v \sqrt{2gH} = 0.985 \times \sqrt{2 \times 9.81 \times 400}$$

$$V_j = 87.26\text{ m/s}$$

$\phi = 0.45$

2. velocity of wheel (u)

Speed ratio $u = \phi \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 400}$

$$u = 39.86\text{ m/s}$$

3. Mean (or) pitch diameter of wheel [D]

$$u = \frac{\pi D N}{60} \Rightarrow u \times 60 = \pi D N$$

$$D = \frac{u \times 60}{\pi N}$$

$$D = \frac{4 \times 60}{\pi \times N} = \frac{39.86 \times 60}{\pi \times 750}$$

$$D = 1.015 \text{ m} \quad D = 1015 \text{ mm}$$

4. Diameter of jets:

$$\frac{d}{D} = \frac{1}{6} \Rightarrow d = \frac{D}{6} = \frac{1.015}{6}$$

$$d = 0.169 \text{ m} \Rightarrow d = 169.19 \text{ mm}$$

5. Jet ratio (m):

$$m = \frac{D}{d} = \frac{1.015}{0.169} = 6$$

6. Bucket dimensions

$$\text{width of the buckets (B)} = 5d \Rightarrow 5 \times 0.169 = 0.845 \text{ m}$$

$$\text{depth: " (T)} = 0.8d \text{ to } 1.2d$$

$$= 1.2d = 1.2 \times 0.169$$

$$T = 0.2028 \text{ m}$$

$$\text{Radial Length (L)} = 2d \text{ to } 3d \Rightarrow 2d = 2 \times 0.169$$

$$L = 0.338 \text{ m}$$

7. no. of buckets (Z)

$$Z = \frac{D}{2d} + 15 \Rightarrow \frac{1.015}{2 \times 0.169} + 15 = 18$$

$$Z = 18 \text{ buckets}$$

8. Number of jets:

$$\frac{Q}{Z} = \frac{\text{Total discharge}}{\text{discharge of one jet}}$$

$$\eta_o = \frac{S.P}{W.P} = \frac{12,110 \times 10^3 \text{ W}}{\left(\frac{eg Q H}{1000}\right) \text{ kW}} = \frac{12,110 \times 10^3 \text{ W}}{9 Q H \text{ kW}}$$

$$0.86 = \frac{12,110 \times 1000}{eg Q H} = \frac{12,110 \times 1000}{1000 \times 9.81 \times Q \times 400}$$

$$Q = \frac{12,110 \times 1000}{9.81 \times 0.86 \times 400}$$

$$Q = 3.5885 \text{ m}^3/\text{s}$$

$$Q = A \times V = \text{area of jet} \times \text{velocity of jet}$$

$$Q = \frac{\pi}{4} \times (0.169)^2 \times 87.26$$

$$A = \frac{\pi}{4} d^2$$

$$Q = 1.9573 \text{ m}^3/\text{s}$$

$$\text{No. of jets} = \frac{Q}{Q} = \frac{3.5885}{1.9573} = 1.833 \approx 2 \text{ jets}$$

Pelton wheel turbine

- 1) The water jet in a pelton wheel is 8cm in dia. and has velocity of 93 m/s. The rotation speed of the wheel is 600 rpm and the deflection angle of the jet is 170° . If the speed ratio is 0.47, determine (i) dia. of wheel (ii) power developed.

Given data $d = 8 \text{ cm} = 0.08 \text{ m}$ To find ① dia. of wheel [D]
 ② Power developed [P]

$$V_1 = 93 \text{ m/s}$$

$$N = 600 \text{ rpm}$$

$$\phi \text{ or } K_u = 0.47$$

$$\text{Angle of deflection} = 170^\circ$$

Solution: $u = \frac{\pi D N}{60}$

Speed Ratio, $K_u = \frac{u}{V}$

$$u = K_u \times V = 0.47 \times 93$$

$$u = 43.71 \text{ m/s}$$

$$u = \frac{\pi D N}{60} \Rightarrow u \times 60 = \pi D N$$

$$\frac{u \times 60}{\pi N} = D \Rightarrow \frac{43.71 \times 60}{\pi \times 60}$$

$$D = 1.391 \text{ mm}$$

$$\text{angle of deflection is } 170^\circ$$

$$\text{Blade angle } \phi = 180^\circ - 170^\circ = 10^\circ$$

$$\sin \theta = \frac{op}{hy}$$

$$\cos \theta = \frac{adj}{hy}$$

$$\tan \theta = \frac{op}{adj}$$

Power (or) work done by the jet on the runner / second (P)

$$F_x = \rho a V_1 (V_{w1} + V_{w2})$$

$$P = F_x \times u$$

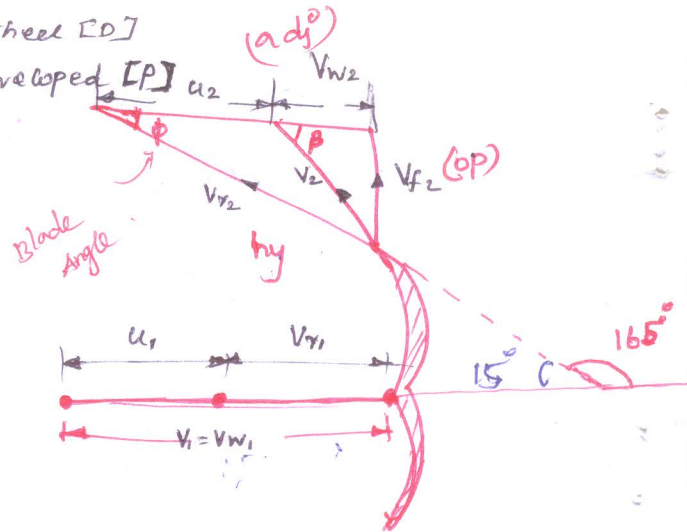
$$P = \rho a V_1 (V_{w1} + V_{w2}) u$$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.08)^2$$

$$a = 5.0265 \times 10^{-3} \text{ m}^2$$

$$u_1 = u_2 = u$$

$$V = \sqrt{2gh}$$



$$V_{w1} = V_1 = 93 \text{ m/s} \text{ given}$$

To find V_{w2}

$$\cos \phi = \frac{adj}{hy}$$

$$\cos \phi = \frac{u_2 + V_{w2}}{V_{r2}}$$

$$\cos \phi V_{r2} = u_2 + V_{w2}$$

$$\cos \phi V_{r2} - u_2 = V_{w2}$$

$$\cos(10^\circ) \times$$

$$V_{r1} = V_1 - u_1$$

$$= 93 - 43.71$$

$$V_{r1} = 49.29 \text{ m/s} = V_{r2}$$

$$\cos(10^\circ) \times 49.29 - 43.71 = V_{w2}$$

$$V_{w2} = 4.83 \text{ m/s}$$

$$P = \rho a V_1 (V_{w1} + V_{w2}) u$$

$$= 1000 \times 5.0265 \times 10^{-3} \times 93 (93 + 4.83) 43.71$$

$$P = 1998.95 \text{ W}$$